Published by Institute of Physics Publishing for SISSA

Received: *January 17, 2008*
Revised: *April 23, 2008*
Accepted: *June 2, 2008*
Published: *June 11, 2008*

iGUT (GUT on interval)

Naoyuki Haba, Yutaka Sakamura and Toshifumi Yamashita

Department of Physics, Osaka University,
Toyonaka, Osaka 560-0043, Japan
E-mail: haba@yukawa.kyoto-u.ac.jp, sakamura@riken.jp,
yamasita@eken.phys.nagoya-u.ac.jp

Abstract: We investigate a construction of five-dimensional (5D) grand unified theories (GUTs) on an interval, which we call iGUTs. We analyze supersymmetric SO(10) iGUT as an example, where the gauge multiplet is spread over the 5D bulk. The SO(10) is directly reduced to the standard model gauge symmetry through the interval boundary conditions. Notice that this rank reduction is impossible in case of GUTs on orbifolds. Four scenarios are possible according to locations (bulk or brane) of Higgs and matter fields. We investigate the gauge-coupling unification, the proton decay, the SO(10) GUT features such as t-b-τ unification and so on in each scenario. We also comment on the flavor phenomenology.

Keywords: GUT, Beyond Standard Model, Gauge Symmetry, Field Theories in Higher Dimensions.

JHEP06(2008)044

Contents

1. Introduction

A supersymmetric (SUSY) grand unified theory (GUT) is an attractive candidate as an underlying theory of the standard model (SM). The strongest reason is that the three SM gauge couplings seem unified at a high energy scale, $\Lambda_G \simeq 2 \times 10^{16}$ GeV, which is the so-called GUT scale. However, recent precise measurements of the QCD gauge coupling show a small but finite deviation from the predicted value of the unification [1]. Also, some theoretical problems exist in the four-dimensional (4D) minimal SUSY SU(5) GUT. For example, the triplet-doublet splitting in the Higgs multiplets should be realized by an unnatural fine-tuning of $\mathcal{O}(10^{14})$. So people pay attention to five-dimensional (5D) SU(5) GUT on an orbifold [2, 3], which realizes the gauge symmetry breaking and the triplet-doublet splitting simultaneously through boundary conditions (BCs) of the orbifold. We do not need to introduce adjoint Higgs fields to break the GUT gauge symmetry which usually violate the R-symmetry explicitly in the superpotential. Furthermore, a precise gauge coupling unification (GCU) can be realized by taking the compactification scale lower than the GUT scale Λ_G [4]. This situation corresponds to take the triplet Higgs masses lighter than Λ_G in the 4D GUTs, which however causes too rapid proton decay [5]. This problem is avoidable in the 5D setup, since the triplet Higgs fields get heavy masses with their chiral partners without violating the R-symmetry [3]. The R-symmetry is valid to forbid problematic dimension-five operators in general. Thus the 5D GUTs on the orbifold are attractive from these phenomenological points of view. However the rank of the GUT gauge symmetry cannot be reduced on the orbifold BCs.[1] So the GUTs with higher ranks than the SM must have extra remaining gauge symmetries, which should be broken by introducing extra elementary Higgs fields. Thus, if we would like to consider SO(10) GUT, which unifies quarks and leptons in a single multiplet, we must introduce additional GUT-symmetry breaking Higgs fields. For example, in ref. [7], the orbifold BCs break SO(10) into the Pati-Salam gauge group, which is subsequently broken to the SM by vacuum expectation values (VEVs) of additional Higgs fields. Another setup is a six-dimensional spacetime where orbifold BCs break SO(10) to the SM gauge group times an extra U(1) which must be broken by additional Higgs fields again [8]. Anyhow, the existence of extra gauge groups is inevitable in the orbifold GUTs.

Recently, some people consider an interval instead of the orbifold for the compactification space in 5D models [9]. It provides larger class of BCs than the orbifold, which are consistent with the action principle. The tree-level unitarity is also maintained for certain interval BCs [9, 10],[2] part of which can be obtained from the orbifold by introducing non-dynamical Higgs fields (which we call *fake* Higgs fields) on the orbifold boundaries and taking their VEVs to infinity [9, 12]. It is remarkable that the rank of the gauge group is reducible by the interval BCs in contrast to the orbifold. We stress that the interval can take BCs which the orbifold cannot realize. For this reason, the interval is useful for the

[1]Precisely speaking, a rank reduction can be possible in a gauge-Higgs unification scenario [6] which is not considered in this paper.

[2]The unitarity under the orbifold BCs in the flat extra dimension is guaranteed by an equivalence theorem [11].

JHEP06(2008)044

extra-dimensional model building in various contexts. However, most of the works on the interval use the interval BCs in models of the electroweak symmetry breaking, namely, the Higgsless models [9, 13] or the gauge-Higgs unification models [14]. The application of the interval BCs to the GUT-symmetry breaking has not been studied so far, except for the trinification model [15].

In this paper, we investigate a construction of 5D $\mathcal{N}=1$ SUSY GUTs on the interval, which we call iGUTs. The gauge multiplets are set to be spread over the 5D bulk. The rank of the GUT gauge symmetry is reduced through the interval BCs differently from the orbifold. In section 2, we consider SO(10) iGUT, and discuss four scenarios depending on locations of the Higgs and matter fields in the extra dimension (bulk or boundary). The discussion on the GCU in the orbifold GUTs [4] is applied for these scenarios in section 3. In section 4, we review a construction of interval BCs by introducing the *fake* Higgs fields on the boundaries and taking their VEVs to infinity. Useful formulae are collected in appendices. Section 5 is devoted to the summary and discussions.

2. SO(10) iGUT

Let us consider the SO(10) iGUT with the flat metric. In this section, we impose interval BCs by hand at the two end points, $y=0$ and πR, which break SO(10) to the SM gauge symmetry. Here y is the 5th dimensional coordinate, and we call these two end points as branes or boundaries in the following discussions. We should remind that any orbifold BCs cannot realize the direct GUT-symmetry breaking of SO(10) $\rightarrow$ SM. The minimum field content is the gauge multiplet $\mathbf{45_G}$, matter multiplets $\mathbf{16_M}$, and a Higgs multiplet $\mathbf{10_H}$. The doublet Higgs fields of the minimal SUSY SM (MSSM) are contained in $\mathbf{10_H}$. There are no GUT-symmetry breaking Higgs in this field content. The gauge multiplet is spread over the 5D bulk, and the matter and Higgs fields are either bulk or brane fields. Realization of the following BCs by use of the *fake* Higgs fields will be discussed in section 4.

As for the gauge multiplet, we take the Neumann (Dirichlet) BCs for the SM (SO(10)/SM) gauge fields A_μ^a ($A_\mu^{\hat{a}}$) on the $y=\pi R$ boundary, where a ($\hat{a}$) denotes the SM (SO(10)/SM) gauge index. Thus, the gauge symmetry is reduced to the SM one at $y=\pi R$. On the other hand, we take the Neumann BCs for all components of the SO(10) gauge multiplet at $y=0$. Therefore the BCs at both the branes are given as

$$\partial_y A_\mu^a = \partial_y A_\mu^{\hat{a}} = 0, \quad (y=0), \qquad \partial_y A_\mu^a = A_\mu^{\hat{a}} = 0, \quad (y=\pi R). \tag{2.1}$$

The BC at $y=0$ makes all components of A_y heavy. Thus, there are no physical degrees of freedom in A_y, which are absorbed into the longitudinal components of massive gauge fields. So we focus on A_μ in the following discussions. The BCs in eq. (2.1) give the lightest mode of $A_\mu^{\hat{a}}$ a mass of $1/(2R)$, while that of A_μ^a remains massless. It means that the 4D effective theory has the SM gauge symmetry. This is a kind of the Higgsless breaking of the GUT symmetry.

We should determine the locations of the matter and Higgs fields for the discussion of phenomenological issues, such as the triplet-doublet splitting, the proton decay, the GCU,

JHEP06(2008)044

and so on. There are the following four scenarios according to the 5D locations of the MSSM Higgs doublets and matter fields.

2.1 Brane Higgs and brane matter

The first scenario is putting both the Higgs doublets and matter fields on the $y = \pi R$ brane. The gauge symmetry on this brane is already reduced to the SM one, so that SO(10)-incomplete multiplets and SO(10)-breaking interactions can be introduced on it. Some features of the SO(10) GUT are lost in this setup, for example, the t-b-τ unification and unification of the right-handed neutrinos and other matters. And the charge quantization $Q(p^+) = -Q(e^-)$ nor the automatic anomaly cancellation of SO(10) are not guaranteed. This setup seems not so attractive, however, has the following good features. Absence of triplet Higgs fields makes the dangerous dimension-five proton decay operators mediated by them vanish. Dimension-six operators are also absent since the coset space gauge fields $A_\mu^{\hat{a}}$ do not couple to the brane matter fields due to no overlap at $y = \pi R$ brane.[3] As for intrinsic dimension-five operators suppressed by the cutoff scale, they are (almost) forbidden by imposing the (approximate) R-symmetry. Remind that this is impossible in the 4D setup, since the R-symmetry is broken at the GUT scale through the triplet and adjoint Higgs masses.

The R-symmetry is set to be broken only in the hidden (SUSY-breaking) sector. There are the following three options for the location of the hidden sector.

Hidden sector localized on the $y = 0$ brane. The SUSY flavor problem can be solved by the gaugino mediation [17]. Recalling that the gravitino mass is $m_{3/2} \simeq F/M_P$ ($M_P \simeq 1.2 \times 10^{19}$GeV: 4D Planck scale, F: order parameter of SUSY breaking), the gaugino mass is expressed as $M_{1/2} = F/(2\pi R\Lambda_*^2) \simeq m_{3/2} \times (\delta^2/\epsilon)$. Here $\epsilon \equiv \Lambda_*/M_P$ (Λ_*: 5D cutoff scale), and $\delta \equiv 1/\sqrt{2\pi R\Lambda_*}$ is the volume suppression factor, which must be less than one if the 5D description is valid.[4] Since Λ_* is at most the 5D Planck scale M_5 that is related to M_P through $M_P^2 = 2\pi R M_5^3$, these quantities satisfy the following relation.

$$0 < \epsilon \leq \delta < 1, \tag{2.2}$$

where the equality holds when $\Lambda_* = M_5$. The other soft SUSY breaking masses are induced from the gaugino mass through the renormalization group equations (RGEs) though they are small at the compactification scale, and then the SUSY flavor problem is solved [17]. Therefore the soft SUSY masses are of the order of the gaugino mass in the low energy.[5] To be more concrete, in the leading-log approximation, flavor independent soft squared masses are generated through the gaugino loop as

$$\tilde{m}^2 = 8Tg_4^2 M_{1/2}^2 \frac{\ln(M_c/M_{\mathrm{SUSY}})}{16\pi^2} \sim M_{1/2}^2, \tag{2.3}$$

[3]The extra-dimensional components $A_y^{\hat{a}}$ do not appear in the 4D minimal coupling with the brane fields. Although higher-derivative interactions of them which are localized on the branes might be possible [12, 16], we assume their absence in this paper, for simplicity.

[4]We assume that brane-localized couplings normalized by Λ_* are of $\mathcal{O}(1)$, while the bulk gauge coupling constant is somewhat large in order to realize the suitable value of the 4D gauge coupling constants.

[5]The anomaly mediation [18] is effective in the case of heavy gravitino mass with $\delta^2/\epsilon \lesssim 10^{-2}$.

where T is a group factor being of order 1, g_4 is the 4D effective gauge coupling and $M_c \equiv 1/R$ is the compactification scale.

In this scenario, the μ-term is difficult to be induced from the hidden sector. The simplest example of generating μ is to introduce a gauge singlet field on the $y = \pi R$ brane whose VEV becomes the μ-term [19].

Hidden sector localized on the $\boldsymbol{y = \pi R}$ brane. The SUSY flavor problem is revived again as in the 4D GUTs. Thus another flavor-independent SUSY mediation must be introduced and dominate the gravity mediation for the suitable soft SUSY breaking masses.

The μ-term can be induced by a direct coupling between the hidden sector's spurion field X and the Higgs fields as $X^\dagger H_u H_d$ in the Kähler potential [20]. This case tends to realize a large $\mu \sim m_{3/2} \times \epsilon^{-1}$ so that the coupling of $X^\dagger H_u H_d$ should be tuned to be small in order for μ to be the same order as $M_{1/2} \sim m_{3/2} \times (\delta^2/\epsilon)$.[6]

Radion $\boldsymbol{F}$-term. The SUSY breaking can be induced through the radion F-term [21, 22], which is equivalent to the Scherk-Schwarz SUSY breaking [23] in the flat metric [21, 24].[7] The gaugino masses are induced from the radion F-term, which derives all soft SUSY masses through the RGEs as above. The gravitino mass is the same order as the gaugino mass in this setup as $M_{1/2} \sim m_{3/2}$.

The μ-term might be obtained by introducing an extra singlet.[8]

2.2 Brane Higgs and bulk matter

The second scenario is putting the matter fields in the bulk while the doublet Higgs fields remaining on the $y = \pi R$ brane. As in the first option, the anomaly cancellation of SO(10) is not automatic. We denote a matter hypermultiplet $\mathbf{16_M}$ as $(\mathbf{16}, \mathbf{16^c})$, where $\mathbf{16}$ and $\mathbf{16^c}$ correspond to $\mathcal{N} = 1$ SUSY chiral multiplets. We take the BCs as

$$\partial_y \mathbf{16} = \mathbf{16^c} = 0 \tag{2.4}$$

at both boundaries. It is worthwhile to notice that the BCs are compatible with the SO(10) bulk gauge symmetry, in contrast to the orbifold BCs.[9] Because the Higgs fields are localized on the $y = \pi R$ brane, the Yukawa interactions have to be localized on the brane, allowing us to introduce appropriate couplings of the MSSM.

Due to the absence of the triplet Higgs fields, dimension-five proton decay operators induced by them are absent, and the intrinsic dimension-five proton decay operators are suppressed by imposing the (approximate) R-symmetry as in the scenario in section 2.1. On the other hand, the dimension-six proton decay processes mediated by the heavy gauge

---

[6] In a similar way, the coupling of $X^\dagger X H_u H_d$ should be tuned to be small to avoid a large B-parameter [17].

[7] These are also equivalent to putting constant superpotentials in the branes [24, 25].

[8] There is no direct interaction $T^\dagger H_u H_d$ (T: radion) on the branes in the flat metric.

[9] For instance, in the SU(5) orbifold model where the BCs breaks SU(5) to the SM symmetry, D^c and L in the $\bar{\mathbf{5}}$ multiplet must have opposite parities. Thus, it is impossible for both components to serve zero-modes from a single $\bar{\mathbf{5}}$ bulk hypermultiplet, but two multiplets should be introduced [3].

JHEP06(2008)044

bosons exist because the matter fields couple to $A_\mu^{\hat{a}}$ in the bulk.[10] The experimental lower bound on the lightest KK mass for $A_\mu^{\hat{a}}$, which is a half of the compactification scale, $1/(2R)$, is estimated using a formula in ref. [5] as

$$\frac{1}{2R} \geq 3 \times 10^{15}\ \text{GeV} \left(\frac{g_1^{\hat{a}}}{g_4}\right) \left(\frac{\tau_p(p \to e\pi^0)}{1.6 \times 10^{33}\text{yrs}}\right)^{1/4} \left(\frac{|\alpha_H|}{0.01(\text{GeV})^3}\right)^{1/2}, \tag{2.5}$$

where g_4 is the unified gauge coupling constant in the effective 4D theory, $\tau_p(p \to e\pi^0)$ is the lower bound on the proton lifetime whose present value is 1.6×10^{33}yrs [1],[11] and α_H is a constant of a nucleon-to-vacuum matrix element which would be between 0.003 and 0.03 [27]. It should be noticed that the coupling of the n-th KK mode for $A_\mu^{\hat{a}}$, $A_\mu^{\hat{a}(n)}$, to the matter fields is a new parameter indicated as $g_n^{\hat{a}}$, which is calculated as an overlap integral of wave functions of the matter fields and $A_\mu^{\hat{a}(n)}$, and thus depends on the localization of the matter fields. The localization of a bulk matter field can be realized by a parity-odd bulk mass, m, which makes the wave function of the zero mode have an exponential profile, $\exp(my)$. It is straightforward to calculate the overlap integral among two wave functions of the matter fields and $A_\mu^{\hat{a}(n)}$, or that of the zero mode of A_μ^a. Then we obtain the ratio between the former and the latter as

$$\frac{g_n^{\hat{a}}}{g_4} = 2\sqrt{2}\frac{mR\left\{-(-1)^n(2n-1) - 4e^{-2\pi mR}mR\right\}(\coth(\pi mR)+1)}{(2n-1)^2 + (4mR)^2}. \tag{2.6}$$

For instance, the ratio for the lightest mode $A_\mu^{\hat{a}(1)}$ is calculated as $g_1^{\hat{a}}/g_4 = \sqrt{2}$ for the $y = 0$ brane-localized matters ($m \to -\infty$), $g_1^{\hat{a}}/g_4 = 2\sqrt{2}/\pi = 0.90$ for the matter with the flat profile ($m = 0$), and $g_1^{\hat{a}}/g_4 = 0$ for the $y = \pi R$ brane-localized matters ($m \to \infty$). In reality, every $A_\mu^{\hat{a}(n)}$ also mediates the proton decay, though its contribution is suppressed by $(2n-1)^{-2}$ compared to that of $A_\mu^{\hat{a}(1)}$ due to the heavier mass. Summing up those contributions, we find that the effective coupling is given as

$$\left(\frac{g_{\text{eff}}^{\hat{a}}}{g_4}\right)^2 \equiv \sum_n \left(\frac{g_n^{\hat{a}}}{g_4}\right)^2 \frac{(1/2)^2}{(n+1/2)^2} = \pi\frac{2(\cosh(2\pi mR)-1) - \sinh(2\pi mR) + 2\pi mRe^{-2\pi mR}}{32mR\sinh^2(\pi mR)}. \tag{2.7}$$

Then, we effectively have $g_{\text{eff}}^{\hat{a}}/g_4 = \sqrt{3\zeta_R(2)/2} = 1.57$ for the $y = 0$ brane-localized matter, $g_{\text{eff}}^{\hat{a}}/g_4 = \sqrt{15\zeta_R(4)/2\pi^2} = 0.91$ for the matter with the flat profile, and $g_{\text{eff}}^{\hat{a}}/g_4 = 0$ for the $y = \pi R$ brane-localized matter. Here $\zeta_R(x)$ is the Riemann's zeta function.

In this way, the value of $g_{\text{eff}}^{\hat{a}}/g_4$ becomes small when the 1st and 2nd generation wave functions are localized around $y = \pi R$, and then the proton decay is strongly suppressed, while the smallness of these generation masses should be realized by small Yukawa couplings on the brane (or by some mechanism, for example, the Froggatt-Nielsen (FN) mechanism [28]). On the other hand, if we want to reproduce the fermion mass hierarchy by the bulk matter localizations [29], the 1st and 2nd generation matter fields should be localized around the $y = 0$ brane. In this case, the value of $g_{\text{eff}}^{\hat{a}}$, and thus the dimension-six proton

[10] In the orbifold models, such dimension-six operators are absent because D^c and L (Q and (U^c, E^c)) reside in different $\bar{\mathbf{5}}$ ($\mathbf{10}$) multiplets, as mentioned in the footnote 9.

[11] A more stringent bound, 5.3×10^{33}yrs, has been reported in ref. [26].

decay, are enhanced. As will be shown in section 3, the precise GCU might need unknown extra fields in the brane Higgs scenarios, so the compactification scale $1/R$ cannot be determined at the present stage. When this mass is of the order of the GUT scale, the decay rate of the process $p \to e\pi$ is enhanced by a factor 6 compared to the minimal SU(5) model. Anyhow, we should notice that the bulk matter profiles cannot explain all fermion mass hierarchies and flavor mixings only by themselves due to the bulk SO(10)-symmetry.

The SUSY flavor problem is not solved due to the existence of the bulk matter fields. Neither the hidden sector on the $y = 0$ brane nor $y = \pi R$ brane can solve it. The radion F-term also induces the SUSY flavor problem due to the generation dependent bulk matter profiles [30].

In some context it can be solved due to suitable localizations of the matter fields, as analysed in section 2.4, but, in principle, another flavor-independent SUSY mediation must be introduced and dominate the gravity mediation for the suitable soft SUSY breaking parameters.

As for the μ-term, the situation is the same as section 2.1. The interaction $X^\dagger H_u H_d$ can induce the suitable value of μ when the hidden sector is localized on the $y = \pi R$ brane.

2.3 Bulk Higgs and brane matter

The third scenario is putting the $\mathbf{10_H}$ Higgs hypermultiplet in the bulk whereas the matter fields on the $y = \pi R$ brane.[12] As the first option in section 2.1, the introduction of the matter fields on the SO(10)-breaking brane means that the charge quantization nor the automatic anomaly cancellation are no longer guaranteed. Denoting the hypermultiplet $\mathbf{10_H}$ as (H, H^c) (H^c: chiral partner), we take BCs for $\mathbf{10_H}$ as

$$\partial_y H = H^c = 0, \quad (y = 0), \qquad \partial_y H_D = H_T = H^c_D = \partial_y H^c_T = 0, \quad (y = \pi R), \tag{2.8}$$

where $H^{(c)} = (H_T^{(c)}, H_D^{(c)})$ with $H_T^{(c)}$ $(H_D^{(c)})$ being the triplet (doublet) Higgs field. Here we omit an index that labels two different Higgs fields, i.e. one forms the up-type Yukawa interactions and the other does the down-type ones. The triplet-doublet splitting is realized through these BCs similarly to the 5D SU(5) GUT on the orbifold [2].

Again, although the SO(10)-relations such as the t-b-τ unification are lost, appropriate Yukawa interactions and Majorana masses of the right-handed neutrinos can be introduced on the $y = \pi R$ brane. Since the triplet Higgs fields, H^T, have the Dirichlet BC, they do not couple with the brane-localized quarks and leptons. The R-symmetry forbids the dangerous intrinsic dimension-five proton decay operators.

In section 3, we will show the bulk Higgs is preferable for the accurate GCU, where the favorite value of $1/(2R)$ is about of $\mathcal{O}(10^{14})$ GeV. This seems dangerous for the proton decay through the dimension-six operators. Nevertheless, this scenario does not have the dimension-six proton decay processes, as the scenario in section 2.1. Furthermore, this setup can solve the SUSY flavor problem when the hidden sector is localized on the $y = 0$ brane via the gaugino mediation as in section 2.1. A difference here is that the bulk

[12] A case of the matter fields localized on the $y = 0$ brane cannot reproduce the realistic fermion mass spectrum.

Higgs multiplets can also play a role of the SUSY breaking mediator through the (flavor dependent) Yukawa interactions. As the gaugino mass, the SUSY-breaking masses of the Higgs fields, $\tilde{m}_h^2$, exist at the tree level via the contact interactions $X^\dagger X H^\dagger H$. These masses contribute to the flavor violation through the loop effects. Such contributions to the soft squared masses are evaluated in the leading-log approximation as

$$\delta\tilde{m}^2 = 2TY^\dagger Y \tilde{m}_h^2 \frac{\ln(M_c/M_{\rm SUSY})}{16\pi^2}, \tag{2.9}$$

where Y is the Yukawa matrix and T is a group factor to be calculated individually. The patterns of the flavor violations induced by (2.9) are exactly the same as the well-known results in the MSSM plus the right-handed neutrinos [31, 32] with the universal SUSY breaking parameters at the cutoff scale, within the leading-log approximation.

In this case the μ-term is generated through $X^\dagger H_u H_d$ with the same order as the soft SUSY masses, $\mu \sim m_{3/2} \times (\delta^2/\epsilon)$ $(\sim M_{1/2})$ because of the volume suppression factor in the interaction $(X^\dagger H_u H_d)$ similar to the gaugino masses. Therefore this scenario is phenomenologically favorable.[13] To be more precise, since the accurate GCU will require $\delta \sim 1/32$ and $\epsilon \sim 10^{-2}$, which are read off from eq. (3.9), the soft SUSY masses and μ are smaller than the gravitino mass as $0.1 \times m_{3/2}$.

In the bulk Higgs scenario, the μ-term might be also obtained through a non-canonical Kähler potential $\mathcal{K} \ni H_u H_d + h.c.$ on the branes and a vanishing cosmological constant condition. This picks up the SUSY and R-symmetry breaking effects[14] in the supergravity (SUGRA) setup, which is the so-called Giudice-Masiero (GM) mechanism [33]. It might induce a small μ-term as $\mu \sim m_{3/2} \times \delta^2$, while $M_{1/2} \sim m_{3/2} \times (\delta^2/\epsilon)$ or $M_{1/2} \sim m_{3/2}$ for the brane-localized hidden sector or the radion F-term scenario, respectively.[15]

2.4 Bulk Higgs and bulk matter

The fourth scenario is putting both the Higgs and matter fields in the bulk. This scenario guarantees the charge quantization as well as the automatic anomaly cancellation of SO(10).[16] As will be shown in section 3, the bulk Higgs setup is preferable for the accurate GCU.

2.4.1 Proton decay

The dimension-five proton decay operators can be suppressed by the approximate R-symmetry, even though the triplet chiral partner H_T^c couples to the matter fields in this case. It should be noticed that the triplet Higgs components H_T become super-heavy through their R-symmetric KK masses with the chiral partners, H_T^c, instead of R-breaking

[13] We need a tuned coupling of $X^\dagger X H_u H_d$ to avoid a large B-parameter [17]. The radion F-term might also solve the SUSY flavor problem, however, the suitable μ-term is not easily generated in the minimal field content as shown below.

[14] The Peccei-Quinn (PQ) symmetry is also broken by this non-canonical Kähler potential.

[15] In both cases the coupling between the gauge fields and the hidden sector fields must be tuned to be small to realize $\mu \sim M_{1/2}$. In such a case, the anomaly mediation effects should be also taken into account.

[16] The automatic anomaly cancellation is lost if an SO(10)-incomplete multiplets are put on the $y = \pi R$ brane.

JHEP06(2008)044

mixing masses between two H_T's. This is an essence of the existence of the (approximate) R-symmetry, which prevents the dimension-five proton decay processes, as keeping the triplet-doublet splitting [3].

In order to suppress the dimension-six proton decay processes, the 1st and 2nd generations should be localized on the $y = \pi R$ brane, as we have already shown in section 2.2. Let us examine how the proton stability constrains the localization of the matter fields in more concrete. As discussed in Secion 2.2, the localization of the i-th generation is controlled by a kink mass m_i, and we analyse the constraints on the parameters. The effective coupling (2.7) for the 1st generation is constrained according to eq. (2.5). For instance, a value $1/(2R) = 3.6 \times 10^{14}$GeV which is calculated in Secion 3 using the central values insists $g^{\hat{a}}_{\rm eff}/g_4 < 0.12$ for $\alpha_H = 0.01(\mathrm{GeV})^3$ in order to be consistent with $\tau_p(p \to e\pi) > 1.6 \times 10^{33}$years. This constraint is converted into that of the parameter m_1 through eq. (2.7) and we find that $m_1 R > 13.6$. This means that the 1st generation should be strictly localized on the $y = \pi R$ brane, in practice.

In addition, the localization of the 2nd generation is constrained by another decay mode $\tau_p(p \to \nu_\mu K) > 6.7 \times 10^{32}$years, which is induced through $\overline{\mathbf{16}}_1\mathbf{16}_1\overline{\mathbf{16}}_2\mathbf{16}_2$. Now, the effective coupling is given as

$$\sum_n \frac{g^{\hat{a}}_{1,n}}{g_4}\frac{g^{\hat{a}}_{2,n}}{g_4}\frac{1}{(2n+1)^2} = \frac{\pi}{32}\left\{\frac{1}{(m_1+m_2)R} - \frac{e^{-2\pi m_2 R}}{m_1 R} - \frac{e^{-2\pi m_1 R}}{m_2 R}\right.$$
$$\left. + e^{2\pi(m_1+m_2)R}\left(\frac{1}{m_1 R} + \frac{1}{m_2 R} - \frac{1}{(m_1+m_2)R} - 2\pi\right)\right\}$$
$$\times \left\{\coth(\pi m_1 R) + 1\right\}\left\{\coth(\pi m_2 R) + 1\right\}, \tag{2.10}$$

where $g^{\hat{a}}_{i,n}$ is defined by eq. (2.6) with replacing m by m_i $(i = 1, 2)$. Assuming the same constraint (2.5) also for this decay mode, the square root of (2.10) is constrained to be smaller than 0.15, leading to a constraint on m_2. For instance, we have $m_2 R > 4.0$ for $m_1 R = 13.6$. For larger m_1, the constraint on m_2 becomes weaker. In such a case, another constraint from the same decay mode induced by $\bar{\mathbf{1}}\mathbf{6}_2\mathbf{16}_2\bar{\mathbf{1}}\mathbf{6}_2\mathbf{16}_2$ may become dominant through the quark mixing. It constrains the effective coupling (2.7) with the replacement $m \to m_2$. Then we obtain $g^{\hat{a}}_{\rm eff}/g_4 < 0.15\lambda^{-1}$, where λ is the mixing angle between the flavor and the mass eigenstates. If it is given by the CKM mixing, i.e. $\lambda \sim 0.22$, we obtain $m_2 R > 0.5$.

In a similar way, the localization of the 3rd generation is possibly constrained by a similar mode $\tau_p(p \to \nu_\tau K) > 6.7 \times 10^{32}$years through the quark mixing between the 2nd and the 3rd generations. If the mixing is given by the CKM angle, i.e. λ^2, the upper bound on the coupling is enhanced by λ^{-2} compared to that of the 2nd generation, leading to no constraint on m_3.

2.4.2 Yukawa interactions

Due to the 5D $\mathcal{N} = 1$ SUSY in the bulk, the Yukawa interactions cannot be written except on the branes.

There are the following typical three cases for the locations of the three generation matters.

Case A. The 3rd generation is localized around the $y = 0$ brane.

In this case there is a possibility to ensure the SO(10) GUT feature, i.e., the t-b-τ unification through the Yukawa interaction on the $y = 0$ brane. The realistic Yukawa couplings for the 1st and 2nd generations are introduced on the $y = \pi R$ brane where the SO(10) symmetry is broken down to the SM one. We must abandon the possibility to explain the fermion mass hierarchy by the matter localization, and assume hierarchical couplings on the brane.

Because the off-diagonal terms in the Yukawa matrices on the $y = 0$ brane do not contribute to the CKM mixing due to the $\mathrm{SU}(2)_R$ symmetry in SO(10), the source of the mixing should be on the $y = \pi R$ brane. In order to reproduce the 2-3 mixing, the 3rd generation has to have an overlapping with this brane no smaller than λ^2. This means that the t-b-τ unification is typically violated by larger than λ^4.

Case B. All generations are localized around the $y = \pi R$ brane, and the Yukawa interactions are also there.

This situation is similar to the scenario in section 2.3, in which the accurate GCU is realized as keeping the proton stability. However, it looses both the explanation of the fermion mass hierarchy by their profiles and the t-b-τ unification.

Case C. The 3rd generation is localized around the $y = \pi R$ brane, and the 1st and 2nd generations are around the $y = 0$ brane.

In this case, the proton decays too rapidly through the dimension-six processes, which is enhanced for the accurate GCU. This difficulty can be avoided when the background geometry is warped. In the warped background [34], all the KK modes are localized around the $y = \pi R$ brane, and thus a mode localized around the $y = 0$ brane has only a tiny overlap with $A_\mu^{\hat{a}}$, which suppresses the proton decay.

We introduce Yukawa interactions with $\mathcal{O}(1)$ couplings on the SO(10)-breaking $y = \pi R$ brane. In this case, although the t-b-τ unification is lost, there is a possibility to explain the suitable fermion mass hierarchies by the matter profiles [29].

2.4.3 SUSY breaking

In general, due to the existence of the matters in the bulk, the SUSY flavor problem is not solved unless another flavor-independent SUSY-breaking mediation is introduced and becomes dominant. Now, the situation is better because the 1st and 2nd generations are taken away from the $y = 0$ brane to suppress the proton decay via the dimension six operators. Thus, if the hidden sector where SUSY is broken is localized on the $y = 0$ brane, the dangerous contact terms among the hidden sector and the 1st/2nd generation are suppressed. For example, if we set $(m_1, m_2)R = (13.6, 4.0)$ and the 3rd generation localized around the $y = 0$ brane, the contact terms for the scalar soft masses of the 1st and 2nd generations are exponentially suppressed as

$$\delta\tilde{m}^2 \sim \begin{pmatrix} 10^{-37} & 10^{-24} & 10^{-18} \\ 10^{-24} & 10^{-11} & 10^{-5} \\ 10^{-18} & 10^{-5} & 1 \end{pmatrix} \tilde{m}_0^2. \tag{2.11}$$

Thus, we can conclude that non-negligible contact terms can appear only in the $(3,3)$ element, in the flavor basis. In order to evaluate the flavor violation, we have to move to the mass basis. In the case when the mixing is given by the CKM matrix, the tree level off-diagonal elements are given as

$$\delta\tilde{m}^2 \sim \begin{pmatrix} & \lambda^5 & \lambda^3 \\ \lambda^5 & & \lambda^2 \\ \lambda^3 & \lambda^2 & \end{pmatrix} \tilde{m}_0^2. \tag{2.12}$$

The diagonal elements are generated through the gaugino loop as eq. (2.3) in the leading-log approximation. Thus, assuming $\tilde{m}_0 \sim M_{1/2}$, we can see that the off-diagonal elements (2.12) give interesting predictions just around the present bounds, calculated in ref. [35] for $M_{\rm SUSY} \sim 350$GeV and not so large $\tan\beta$. Now, $\tan\beta$ is large to realize the t-b-τ unification, and thus the bounds cannot be applied as they are in the reference. Nevertheless, this observation is useful to get a rough sketch whether the contact terms are crucially dangerous or not. The actual bounds in this model would be revealed by a more detailed analysis using the full RGEs, which is one of our future works.

As for the μ-term, the situation is the same as section 2.3, where the direct interaction $X^\dagger H_u H_d$ on the brane works well. Also, the scalar masses of the Higgs fields exist via $X^\dagger X H^\dagger H$, and contribute to the flavor violation through the loop effects as evaluated in eq. (2.9), giving a similar contributions as in the MSSM plus the right-handed neutrinos with the universal soft terms.

3. Gauge coupling unification

Since higher dimensional gauge theories are non-renormalizable, it is not easy to trace the flow of each gauge coupling constant above the compactification scale. Nevertheless it is known that, if there is the unified symmetry in the bulk, flows of differences of two different gauge coupling constants, $\delta\alpha_i^{-1} \equiv \alpha_i^{-1} - \alpha_1^{-1}$, are at most logarithmic in the orbifold models [3, 4, 12]. Thus we can examine whether the three gauge couplings are unified or not. Essentially the same discussion can be also applied to the iGUTs, and we show it in the following.

It is convenient to introduce the following non-analytic but continuous function for $x \geq 1$:

$$f(x) = \sum_{k=1}^{k=k_x-1} (-1)^k \ln\left(\frac{k+1}{k}\right) + (-1)^{k_x} \ln\left(\frac{x}{k_x}\right), \tag{3.1}$$

where k_x is the natural number that satisfies $x-1 \leq k_x < x$. This function converges for large x as $f(\infty) = -\ln(\pi/2) \sim -0.45$. In the following analysis, we approximate this function by $f(\infty)$ for $x \gtrsim 10$, because an error induced by this approximation is of $\mathcal{O}(1/(2x))$.

First, let us evaluate the contributions from the bulk $\mathbf{10_H}$ Higgs hypermultiplet with BCs in eq. (2.8) above the (half of) compactification scale. Here we do not introduce parity-odd bulk masses, for simplicity. The KK spectra of the doublets and triplets are n/R and $(n+1/2)/R$, respectively. A pair of the doublet and triplet compose a (full) multiplet of

SU(5), and then the contribution from this pair is common to the flow of each coupling. This means that a triplet contributes to the flows of $\delta\alpha_i^{-1}$ by the same factor as a doublet but with the opposite sign. In each KK state, there are four doublets or four triplets except for the zero-modes. (The zero-modes consist of only the two Higgs doublets.) Therefore the contribution of the $\mathbf{10_H}$ hypermultiplet above $1/(2R)$ is given by

$$\Delta_H \delta\alpha_i^{-1}(\mu) = -2\frac{\delta C_D^i}{2\pi} f\left(2R\mu\right) \sim \frac{\delta C_D^i}{\pi} \ln\left(\frac{\pi}{2}\right), \tag{3.2}$$

where $\delta C_D = (0, 1/5, -3/10)$ is the contribution by the Higgs doublet to the flow of $\delta\alpha_i^{-1}$. On the other hand, the contribution from the two Higgs-doublet superfields localized on the brane is given by

$$\Delta_H \delta\alpha_i^{-1}(\mu) = -\frac{\delta C_D^i}{\pi} \ln\left(2R\mu\right), \tag{3.3}$$

as in the usual 4D models. Comparing eqs. (3.2) and (3.3), we notice that the sign is flipped when the Higgs multiplets start propagating in the bulk.

Next we evaluate the contributions from the gauge multiplet with the BCs in eq. (2.1). The KK modes with a mass n/R in A_μ^a compose an SO(10) multiplet together with those with a mass $(n+1/2)/R$ in $A_\mu^{\hat{a}}$, so that the former contributes to the flows of $\delta\alpha_i^{-1}$ by the same factor as the latter but with the opposite sign. Since a massless vector (chiral) supermultiplet contributes -3 (1), a massive vector supermultiplet contributes $-3+1=-2$. Thus, above $1/(2R)$, the SO(10) multiplet $(A_\mu^a, A_\mu^{\hat{a}})$ contributes to the flow of $\delta\alpha_i^{-1}$ as

$$\Delta_g \delta\alpha_i^{-1}(\mu) = -\frac{\delta C_g^i}{2\pi}\left(-2\ln\left(2R\mu\right) - f\left(2R\mu\right)\right) \sim -\frac{\delta C_g^i}{2\pi}\left(-2\ln\left(2R\mu\right) + \ln\left(\frac{\pi}{2}\right)\right), \tag{3.4}$$

where $\delta C_g = (0, 2, 3)$ is the contribution from the MSSM gauge sector.

As for the matter fields, they do not contribute to the flows of $\delta\alpha_i^{-1}$ because they compose degenerate SO(10) full multiplets. Then, in summary, we obtain

$$\delta\alpha_i^{-1}(\mu) \sim \delta\alpha_i^{-1}\left(\frac{1}{2R}\right) - \frac{\delta C_g^i}{2\pi}\left(-2\ln\left(2R\mu\right) + \ln\left(\frac{\pi}{2}\right)\right) + \Delta_H \delta\alpha_i^{-1}(\mu). \tag{3.5}$$

Defining $\Lambda_{\rm G}$ by $\delta\alpha_2^{-1}(\Lambda_{\rm G}) = 0$ in the MSSM, the value of $\alpha_i^{-1}\left(1/(2R)\right)$ is determined as

$$\delta\alpha_i^{-1}\left(\frac{1}{2R}\right) = \delta\alpha_i^{-1}(\Lambda_{\rm G}) + \frac{\delta b_i}{2\pi}\ln\left(2R\Lambda_{\rm G}\right), \tag{3.6}$$

where $\delta b_i = (0, -28/5, -48/5)$ is the difference of the beta functions in the MSSM.

Now we can estimate the deviations from the MSSM, depending on the Higgs profiles. We determine the value of Λ_* by use of $\delta\alpha_2^{-1}(\Lambda_*) = 0$. By imposing $\delta\alpha_3^{-1}(\Lambda_*) = 0$, we determine $1/(2R)$ and Λ_* as a function of $\delta\alpha_3^{-1}(\Lambda_{\rm G})$, which is the problematic disagreement of the QCD coupling in the 4D minimal SU(5) GUT. Neglecting the GUT threshold correction, the deviation is estimated as $\delta\alpha_3^{-1}(\Lambda_{\rm G}) = 0.855 \pm 0.315$ [4]. Anyhow, the GCU crucially depends on whether the Higgs fields are located in the bulk or on the brane, so we analyze the GCU in each case.

JHEP06(2008)044

Bulk Higgs case

Equation (3.2) derives

$$\delta\alpha_i^{-1}(\mu) \sim \delta\alpha_i^{-1}\left(\frac{1}{2R}\right) - \frac{\delta C_g^i}{2\pi}\left(-2\ln(2R\mu) + \ln\left(\frac{\pi}{2}\right)\right) + \frac{\delta C_D^i}{\pi}\ln\left(\frac{\pi}{2}\right). \tag{3.7}$$

Taking $\delta\alpha_i^{-1}(\Lambda_*) = 0$, we can calculate $(\ln(2R\Lambda_{\rm G}), \ln(2R\Lambda_*))$ as

$$\begin{pmatrix} \ln(2R\Lambda_{\rm G}) \\ \ln(2R\Lambda_*) \end{pmatrix} = \begin{pmatrix} \frac{1}{3}2\pi\delta\alpha_3^{-1}(\Lambda_{\rm G}) - \ln\left(\frac{\pi}{2}\right) \\ \frac{7}{15}2\pi\delta\alpha_3^{-1}(\Lambda_{\rm G}) - \ln\left(\frac{\pi}{2}\right) \end{pmatrix} = \begin{pmatrix} 4.00 \\ 5.78 \end{pmatrix} \tag{3.8}$$

for $\delta\alpha_3^{-1}(\Lambda_{\rm G}) = 0.855$. This means

$$(2R\Lambda_{\rm G},\ 2R\Lambda_*) = (55,\ 320), \tag{3.9}$$

which is consistent with refs. [4]. In this case, the mass of the lightest modes in $A_\mu^{\hat{a}}$ is evaluated as $1/(2R) = 3.6\times 10^{14}$ GeV, which is too light to be consistent with the proton decay constraint unless the coupling $g_{\rm eff}^{\hat{a}}$ is small as $g_{\rm eff}^{\hat{a}}/g_4 < 0.12$ (0.21) for $\alpha_H = 0.01$ (0.003).[17] It can be achieved when the 1st generation matter is localized around $y = \pi R$. (A typical case is $g_{\rm eff}^{\hat{a}} = 0$ which corresponds to the matters strictly localized on the $y = \pi R$ brane.) This constraint plays a crucial role for the construction of models as shown in section 2.4.

Brane Higgs case

By similar calculations, we obtain

$$\begin{pmatrix} \ln(2R\Lambda_{\rm G}) \\ \ln(2R\Lambda_*) \end{pmatrix} = \begin{pmatrix} -8.5 \\ -13 \end{pmatrix} \tag{3.10}$$

by use of eq. (3.3). However, this means $\Lambda_* < \Lambda_{\rm G} < 1/(2R)$, a nonsense relation. This implies the precise GCU is difficult in the brane Higgs scenario. Thus, introduction of extra SO(10) incomplete multiplets on the $y = \pi R$ brane might be required for the precise GCU.

Recalling that the light triplet Higgs multiplets are preferred for the GCU in the 4D minimal GUT [5], the bulk Higgs scenario, which can have the light triplets, might be preferred than the brane Higgs scenario. (We should emphasize again that the light triplets in the 4D GUT induces too rapid proton decay.)

4. Interval BCs by *fake* Higgs

Some of the interval BCs can be obtained from an orbifold S^1/Z_2 by a method which we call the *fake* Higgs construction. In this paper we focus on such types of BCs, which are expected to be consistent with the tree-level unitarity and the Ward-Takahashi identities [9, 10]. The *fake* Higgs construction of the interval BCs was first introduced in ref. [12]. For reader's convenience, we review this method in this section. We discuss general arguments first, and then give the SO(10) BCs on an interval.

[17] For the smaller value by 1-σ, $\delta\alpha_3^{-1}(\Lambda_{\rm G}) = 0.539$, the gauge boson mass is modified as $1/(2R) = 1.9\times 10^{15}$GeV, which still requires a little bit small $g_{\rm eff}^{\hat{a}}$ or $|\alpha_H|$ as $g_{\rm eff}^{\hat{a}}/g_4 < 0.60$ (1.1) for $\alpha_H = 0.01$ (0.003).

JHEP06(2008)044

4.1 General arguments

In the orbifold, BCs are strictly restricted by the orbifolding parity if there are no boundary terms. Namely, fields with even (odd) parities follow the Neumann (Dirichlet) BCs automatically. However, in the interval, the even (odd) parity does not automatically correspond to the Neumann (Dirichlet) BC. Thus, more general BCs are possible on the interval, which broaden the possibility of the model-building. Some of them are obtained by introducing 4D scalar fields on the boundaries, whose VEVs break part of the residual symmetries under the orbifold projection, and taking their VEVs to infinity. We name such boundary fields as *fake* Higgs fields because they are not dynamical degrees of freedom after taking the limit. The effects of the boundary Higgs fields are replaced by the boundary masses after they get VEVs. The detailed calculations are provided in appendices. In this subsection we will explicitly see how the boundary masses change the mass spectra and BCs of the bulk fields in some simple examples to illustrate the situation.

4.1.1 Gauge sector

Here we consider a case that part of the gauge symmetries is broken at $y = \pi R$ by the boundary masses $\mathcal{M}_{\hat{a}}$ for the gauge fields $A_\mu^{\hat{a}}$, which are induced by the VEVs of the boundary *fake* Higgs fields. The mass spectrum is determined by eq. (A.19) in appendix A.1. In the flat spacetime, it becomes

$$\tan(m_{\hat{a},n}\pi R) = \frac{\mathcal{M}_{\hat{a}}}{2m_{\hat{a},n}}, \tag{4.1}$$

and the mode functions (profiles of wave functions) are given by

$$f_n^{\hat{a}}(y) = \left(\frac{\pi R}{2} + \frac{\mathcal{M}_{\hat{a}}}{4m_{\hat{a},n}^2 + \mathcal{M}_{\hat{a}}^2}\right)^{-1/2} \cos(m_{\hat{a},n} y), \tag{4.2}$$

where $m_{\hat{a},n}$ are solutions of eq. (4.1).

In the case of no boundary mass, i.e., $\mathcal{M}_{\hat{a}} = 0$, the gauge field $A_\mu^{\hat{a}}$ follows the Neumann BCs at both boundaries and the mass eigenvalues are $m_{\hat{a},n} = n/R$ (n: integer), which is just the case of the orbifold. If we turn on the boundary mass $\mathcal{M}_{\hat{a}}$, the eigenvalues are shifted as

$$m_{\hat{a},n} = \frac{n}{R} + \frac{1}{\pi R}\arctan\left(\frac{\mathcal{M}_{\hat{a}}}{2m_{\hat{a},n}}\right). \tag{4.3}$$

For a finite $\mathcal{M}_{\hat{a}}$, the shift of the mass eigenvalue monotonically decreases as the KK level n increases, and becomes negligible for $m_{\hat{a},n} \gg \mathcal{M}_{\hat{a}}$. In the limit of $\mathcal{M}_{\hat{a}} \to \infty$, on the other hand, all eigenvalues are uniformly shifted by $1/(2R)$, which indicates that the boundary condition at $y = \pi R$ changes from Neumann to Dirichlet. This can be seen explicitly from eq. (4.2). The boundary value of the mode function at $y = \pi R$ is given by

$$\left|f_n^{\hat{a}}(\pi R)\right| = \left(\frac{\pi R}{2} + \frac{\mathcal{M}_{\hat{a}}}{4m_{\hat{a},n}^2 + \mathcal{M}_{\hat{a}}^2}\right)^{-1/2} \left|\frac{2m_{\hat{a},n}}{\sqrt{4m_{\hat{a},n}^2 + \mathcal{M}_{\hat{a}}^2}}\right|, \tag{4.4}$$

by using eq. (4.1). In the limit of $\mathcal{M}_{\hat{a}} \to \infty$, this goes down to zero, i.e., $f_n^{\hat{a}}(y)$ follows the Dirichlet BC at $y = \pi R$. We should remember that the parity eigenvalues never change at any BCs realized by the *fake* Higgs.

4.1.2 Hypermultiplet sector

Next we see mass spectra of hypermultiplets in the presence of boundary masses. In appendix A.2.1, we consider a case that the bulk hypermultiplets have mass terms localized at $y = \pi R$. Here let us focus on a case that two hypermultiplets have only the boundary Dirac mass η in a flat spacetime,[18] for simplicity. We take the orbifold parities of the hypermultiplets as eq. (A.24). In this case, eq. (A.37) is reduced to

$$\tan^2(m_n \pi R) = |\eta|^2 , \tag{4.5}$$

where m_n is a mass eigenvalue of the n-th KK mode. The solution of eq. (4.5) is given by

$$m_n = \frac{n}{R} \pm \frac{\arctan|\eta|}{\pi R}. \tag{4.6}$$

It should be noticed that all mass eigenvalues receive the same shift due to the boundary mass η independently of the KK level n, even for finite η. This is in contrast to the case of the gauge sector in eq. (4.3). In the limit of $|\eta| \to \infty$, the shift of the mass eigenvalues becomes $1/(2R)$, which means that BC of even-parity fields at $y = \pi R$ changes from Neumann to Dirichlet. The mode functions defined in eq. (A.29) are (for $0 < y < \pi R$) given as

$$\begin{aligned} f_{h,n}(y) &= \alpha_{h,n} \cos(m_n y), & f_{H,n}(y) &= \pm \frac{\eta^*}{|\eta|} \alpha^*_{h,n} \cos(m_n y), \\ f^c_{h,n}(y) &= -\alpha^*_{h,n} \sin(m_n y), & f^c_{H,n}(y) &= \mp \frac{\eta}{|\eta|} \alpha_{h,n} \sin(m_n y), \end{aligned} \tag{4.7}$$

where the double signs correspond to that in eq. (4.6), and the complex constants $\alpha_{h,n}$'s are determined by the normalization condition, eq. (A.39). Again, we should remember that the parity eigenvalues do not change even when BCs change.

When we take orbifold parities as eq. (A.44), eq. (4.5) is modified as

$$\cot^2(m_n \pi R) = |\eta|^2 , \tag{4.8}$$

where the mass spectrum is given by

$$m_n = \frac{n + \frac{1}{2}}{R} \pm \frac{\arctan|\eta|}{\pi R}. \tag{4.9}$$

It means that the shift of the eigenvalues by η is the same as that in eq. (4.6). Notice that no zero-mode exists when $\eta = 0$, however, it appears in the limit of $|\eta| \to \infty$. This indicates that BC of h at $y = \pi R$ changes from Dirichlet to Neumann. The mode functions are the same as eq. (4.7), but m_n in the arguments are now given by eq. (4.9) and the double signs correspond to that in it.

Note that BCs of the mode functions $f_{\phi,n}$ and $f^c_{\phi,n}$ ($\phi = h, H$) are related to each other by the bulk mode equations in eq. (A.30). Therefore if BC of a chiral multiplet changes from Neumann to Dirichlet, that of its chiral partner inevitably changes from Dirichlet to

[18] The parameter η is dimensionless, which corresponds to a ratio of the *fake* Higgs VEV to the 5D cutoff scale Λ_*.

Neumann. Since the orbifold parities are unchanged by the boundary terms, the mode function of a parity-odd field becomes discontinuous at the boundary when BC changes from Dirichlet to Neumann.

A similar relation exists between V^A and Φ^A ($A = a, \hat{a}$) in the gauge multiplet. As will be mentioned in appendix A.1, the gauge-scalar multiplet Φ^A is absorbed into the $\mathcal{N} = 1$ vector multiplet $\partial_y V^a$. This means that the mode functions for the former are the same as the derivative of the mode functions for the latter. On the other hand, the mode equation for A_y^A is decoupled from A_μ^A by choosing a particular gauge-fixing function. So the mode functions and KK spectrum for A_y^A are independent of the boundary masses for A_μ^A. (*See, for example*, ref. [36].) It seems contradict with the above relation between V^A and Φ^A. However, we should remind that A_y^A is unphysical degree of freedom because it is eaten by A_μ^A through the "Higgs mechanism". In fact the mode function and the spectrum for A_y^A are gauge-dependent. Thus, we can always choose a gauge-fixing function for the KK spectrum of A_y^A to coincide with that of A_μ^A (except for the zero-mode). The $\mathcal{N} = 1$ superfield description in this paper corresponds to this gauge.

When we take orbifold parities as eq. (A.40), situation is quite different from the previous two cases. Equation (A.43) is reduced to

$$\sin(m_n \pi R) \cos(m_n \pi R) = 0, \tag{4.10}$$

which means that the spectrum $m_n = n/(2R)$ is unchanged by the boundary mass η. Thus the zero-mode always exists irrespectively of the value of η. The mode functions have explicit $|\eta|$-dependence (for $0 < y < \pi R$) as

$$\begin{aligned} f_{h,n}(y) &= \alpha_{h,n} \cos(m_n y), & f_{H,n}(y) &= \alpha_{H,n} \cos(m_n y), \\ f_{h,n}^c(y) &= -\alpha_{h,n}^* \sin(m_n y), & f_{H,n}^c(y) &= -\alpha_{H,n}^* \sin(m_n y), \end{aligned} \tag{4.11}$$

where

$$\alpha_{H,n} = \begin{cases} -\frac{1}{\eta} \alpha_{h,n}^* & (n\text{: even}), \\ \eta^* \alpha_{h,n}^* & (n\text{: odd}). \end{cases} \tag{4.12}$$

This is in contrast to the previous cases, where $|\eta|$-dependence of the mode function appears only through the mass eigenvalue. For $\sin(m_n \pi R) = 0$ (i.e., n is even), for example, the modes reside only in (H, H^c) when $\eta = 0$. Equation (4.12) means that this mode continuously moves from (H, H^c) to (h, h^c) as $|\eta|$ increases. We can also infer this behavior from the fact that BCs are interchanged between the two hypermultiplets when $|\eta|$ goes from zero to infinity.

Finally we consider a mixing mass between a bulk hypermultiplet (H, H^c) and a chiral multiplet χ localized on the $y = \pi R$ brane. Here we focus on a simple case that a bulk mass term is absent and the spacetime is flat. Then eq. (A.56) is reduced to

$$\tan(m_n \pi R) = \frac{|\xi|^2}{2(m_n \pm |m_\chi|)}, \tag{4.13}$$

for the parity assignment of eq. (A.48), and

$$\cot(m_n \pi R) = -\frac{|\xi|^2}{2(m_n \pm |m_\chi|)}, \tag{4.14}$$

for the parity assignment of eq. (A.57). The mixing parameter ξ has mass-dimension 1/2 and the mass parameter for χ, m_χ, has mass-dimension 1. (*See* eq. (A.49).) Equation (4.13) has the same forms as eq. (4.1) if we replace $|\xi|^2$ with $\mathcal{M}_{\hat{a}}$ and set $m_\chi = 0$. Thus the $|\xi|$-dependence of the spectrum is similar to that of the gauge multiplet. Due to the existence of the boundary term at $y = \pi R$, the parity-odd field becomes discontinuous there. From eq. (A.51) (or the counterpart in the case of eq. (A.57)), the 4D chiral multiplet χ is expressed as this discontinuity.

4.2 *Fake* Higgs in SO(10) GUT

In section 2 we introduced SO(10) incomplete multiplets and SO(10)-breaking interactions on the $y = \pi R$ brane by hand, since the gauge group is already reduced to the SM gauge symmetry there. In this subsection, we show an explicit realization of this setup by the *fake* Higgs construction. We start from an SO(10)-invariant theory on S^1/Z_2, where A_y has odd-parity so that it has no zero-modes. This means that the charge quantization and anomaly cancellation are ensured in this setup.

In order to obtain the BCs in eq. (2.1), we put $\mathbf{45_H}$, $\mathbf{16_H}$, and $\overline{\mathbf{16}}_\mathbf{H}$ *fake* Higgs fields on the $y = \pi R$ brane. The $\mathbf{45_H}$ Higgs takes a VEV of diag.$(\sigma_2, \sigma_2, \sigma_2, 0, 0)\ v_{\mathbf{45}}$, which reduces the gauge symmetry as $\mathrm{SO}(10) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ at the energy scale of $v_{\mathbf{45}}$, where σ_2 is a Pauli matrix. And the $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ take VEVs in a D-flat direction which lead to the breaking of $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \to \mathrm{U}(1)_Y$. Taking the VEVs of the *fake* Higgs to infinity, the BCs for the gauge multiplet in eq. (2.1) are obtained. The $\mathbf{45_H}$, $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ *fake* Higgs fields are assumed to have suitable interactions among them in order not to leave light (colored) degrees of freedom.[19]

4.2.1 Triplet-doublet splitting

Realization of the triplet-doublet splitting can be achieved by using a technique of Dimopoulos-Wilczek (DW) mechanism [37]. The VEV of the $\mathbf{45_H}$ *fake* Higgs in a direction of $\mathrm{U}(1)_{B-L}$ generator induces the triplet Higgs masses as keeping the doublet Higgs massless. It is justified as far as the doublet Higgs fields are contained in $\mathbf{10_H}$, since the doublets in $\mathbf{10_H}$ have vanishing $\mathrm{U}(1)_{B-L}$ charges. Here we have to introduce additional $\mathbf{10'_H}$ on the $y = \pi R$ brane to allow the coupling between $\mathbf{10_H}$ and $\mathbf{45_H}$. This is because two identical $\mathbf{10_H}$ multiplets cannot form Yukawa interactions with $\mathbf{45_H}$ according to the SO(10) group structure,

$$\mathbf{10} \times \mathbf{10} = \mathbf{1}_S + \mathbf{45}_A + \mathbf{54}_S,$$

where subscript S (A) indicates that the product is (anti-)symmetric. The brane superpotential which realizes the triplet-doublet splitting is given by[20]

$$W_{\mathrm{DW}} = \delta(y - \pi R) \left(y_{\mathrm{DW}} \frac{\mathbf{10_H}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{10_H}}}} \mathbf{45_H} \mathbf{10'_H} + m_{\mathrm{DW}} \mathbf{10'_H}^2 \right), \tag{4.15}$$

[19] If these interactions are absent, components with $(3,2)_{1/6}$ and $(3^*,2)_{-2/3}$ for $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ become pseudo-NG bosons even in the limit of infinite VEVs. (If gauge interactions are switched off, they become exact NG bosons.)

[20] Here we assume that terms such as $\mathbf{10_H}^2$ which destroy the DW mechanism are absent.

where $\delta_{\mathbf{10_H}} = 1(0)$ for bulk (brane) $\mathbf{10_H}$ field. It is natural to regard $\mathbf{10'_H}$ as a *fake* Higgs too, so that $m_{\rm DW}$ should be taken to infinity. When $\mathbf{10_H}$ is a brane field, only the MSSM doublet Higgs components remain to be massless and other fields decouple by getting super-heavy with large masses $y_{\rm DW} v_{\mathbf{45}}$ and $m_{\rm DW}$. When $\mathbf{10_H}$ is a bulk field, the KK spectrum is given as eq. (4.13) by identifying ξ and m_χ with $y_{\rm DW}\langle \mathbf{45_H}\rangle/\sqrt{\Lambda_*}$ and $m_{\rm DW}$, respectively. Thus, for the triplet components, the lightest KK modes obtain masses of $\mathcal{O}(1/(2R))$, while the doublet components remain to be massless, which do not couple to $\mathbf{45_H}$.

4.2.2 Brane Interactions

Here we comment on an idea of taking zero limits of the *fake* Higgs couplings in order to obtain the finite matter interactions and masses effectively. We know that the wrong GUT relations of the mass spectra between the down-type quarks and charged leptons can be modified by the effects of SU(5)-breaking VEVs. The realistic Yukawa matrices might be induced from the brane interactions,

$$\delta(y-\pi R) Y_{n,m} \frac{\mathbf{16}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{16}}}} \frac{\mathbf{16}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{16}}}} \frac{\mathbf{10_H}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{10_H}}}} \left(\frac{\mathbf{45_H}}{\Lambda_*}\right)^n \left(\frac{\mathbf{16_H}\overline{\mathbf{16}}_\mathbf{H}}{\Lambda_*^2}\right)^m, \tag{4.16}$$

where $\delta_{\mathbf{16}} = 1(0)$ for bulk (brane) $\mathbf{16}$ matter. The lowest order, $n = m = 0$, gives an SO(10)-symmetric Yukawa coupling.[21] This is an example of the FN mechanism. The effective Yukawa couplings are divergent by the infinite VEVs of the *fake* Higgs for the finite magnitude of $Y_{n,m}$ $(n, m > 0)$. So, in order to obtain finite Yukawa couplings from eq. (4.16), the couplings $Y_{n,m}$ must be infinitely small as keeping $Y_{n,m}\langle \mathbf{45_H}\rangle^n(\langle \mathbf{16_H}\rangle\langle \overline{\mathbf{16}}_\mathbf{H}\rangle)^m$ finite. The finite Majorana masses of the right-handed neutrinos might be also obtained from the brane interaction,

$$\delta(y-\pi R)\omega \frac{\mathbf{16}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{16}}}} \frac{\mathbf{16}}{\sqrt{\Lambda_*}^{\delta_{\mathbf{16}}}} \frac{\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}}{\Lambda_*}, \tag{4.17}$$

where a coupling ω should be tuned for the suitable magnitudes of Majorana masses.[22]

5. Summary and discussion

We have discussed 5D SUSY GUTs on the interval, where the gauge multiplets propagate in the 5D bulk. Interval BCs make the rank reduction of the gauge symmetry possible in contrast to the orbifold BCs. Although this idea of the rank reduction by BCs is well-known [9], most models use it to break the electro-weak symmetry [13] but the application to the GUT breaking has not been studied except for the trinification model [15].

We have investigated the 5D SO(10) iGUT, in which the gauge symmetry is directly reduced to the SM without introducing GUT-breaking Higgs fields. This is in contrast to the orbifold GUTs where the rank reduction is impossible. We can also consider iGUTs based on other higher-rank gauge symmetries, such as E_6.

[21] If $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ do not couple to the matter fields, which means $m = 0$, the CKM mixing angles vanish because of the $\mathrm{SU}(2)_R$ symmetry which commutes with $\langle \mathbf{45_H}\rangle$.

[22] The KK masses do not break the lepton number.

To be more concrete, we investigated the GCU, the proton decay and the SO(10) features such as t-b-τ unification and charge quantization for different localization of the matter and Higgs fields. We also estimated the flavor violations by the SUSY partners. We briefly summarize our results:

1. *Bulk Higgs scenario.*
   The GCU is improved, i.e., the small disagreement of the QCD coupling from the predicted value in the 4D GCU can be corrected by the existence of the light triplet Higgs modes. For this purpose, a compactification scale lower than the GUT scale is required, demanding the matter fields to be localized around the $y = \pi R$ brane for the proton stability.

   Because the bulk Higgs fields can couple to the SUSY breaking sector, the mu term can be induced through a contact term. In a similar way, the scalar soft squared masses for the Higgs fields can be generated and then induces the flavor violations via the RGE effects, which is similar to that in the MSSM with the right-handed neutrinos.

2. *Brane Higgs scenario.*
   We can introduce only the doublet components of the physical Higgs on the SO(10)-breaking brane. This means that there is no dimension-five proton decay operators induced by the triplet Higgses. Additional SO(10)-incomplete multiplets might be needed for realizing the precise GCU.

   In order to realize an appropriate μ term, some additional mechanism such as the NMSSM may be required.

3. *Bulk matter scenario.*
   The charge quantization of $Q(p^+) = -Q(e^-)$ is ensured. If the 3rd generation matter field is localized around the SO(10)-preserving brane, the t-b-τ unification can be also realized.

   Since bulk matters in general cause the SUSY flavor problem, another source of SUSY breaking may be needed which induces flavor-independent soft masses. When the 1st and 2nd generations are localized around the $y = \pi R$ brane, which is required by the proton decay constraint for the improved GCU, the flavor violations are suppressed. When the 3rd generation has overlapping with the SUSY breaking brane, the contact term generates a sizable contribution to the $(3, 3)$ element of the scalar soft mass matrices at the mediation scale, in the flavor basis. Although the off-diagonal elements are negligible in the flavor basis, the flavor violation can occur through the mixing matrix between the flavor and the mass bases. Especially if this mixing matrix is given by the CKM matrix, the flavor violation is estimated around the experimental bounds.

4. *Brane matter scenario.*
   We loose some of the GUT-predictions such as the charge quantization and the t-b-τ unification.

The SUSY flavor problem can be solved by the sequestering (gaugino mediation), and the dimension-six proton decay processes are absent in this setup.

In each case, all dimension-five proton decay processes can be suppressed by the (approximate) R-symmetry [3]. The realistic Yukawa interactions and the Majorana masses can be reproduced with the help of the superpotential localized on the SO(10)-breaking brane. As for the anomaly cancellation, the automatic cancellation of SO(10) is lost once SO(10)-incomplete multiplets are introduced on the SO(10)-breaking brane. Here, we would emphasize that the couplings between the bulk matter fields and the gauge fields for the broken generators (e.g. the X gauge boson for SU(5) models) are non-vanishing, and induce the proton decay via the dimension-six operators. This is in great contrast with the orbifold GUTs where these couplings are absent because of the constrained parity assignments.

The interval BCs were first considered in ref. [9]. Then, ref. [10] investigates their consistency and finds some BCs that violate the tree-level unitarity and the Ward-Takahashi identities. In order to avoid such dangerous BCs, we used BCs obtained by introducing Higgs fields localized on the boundaries and taking a limit that their VEVs go to infinity [9], which we call the *fake* Higgs construction.

Finally, let us comment on the warped spacetime. The iGUTs can be applied also in the warped 5D background [34]. The equations in the appendices are useful also in the warped setup. We mentioned that the constraints from the dimension-six proton decay are largely modified from the flat case due to the wave-function profiles of the lower KK modes, while the discussion on the symmetry breaking pattern and the location of the hidden sector are not. As for the GCU, we have a technical difficulty in the analysis since the KK mass spectrum cannot be calculated analytically, although it is expected that qualitative features are not drastically changed from the flat case. If the gauge coupling evolution is defined by two-point Green functions of the gauge fields with external lines on the UV brane, it develops logarithmically, and thus is calculable [38]. In this case, the difference of the gauge couplings are frozen out above the IR scale. Namely the GCU is the same as the situation in the MSSM, when the IR scale is the GUT scale.

Acknowledgments

We would like to thank Y. Hosotani for useful discussions. N. H. is supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No.16540258 and No.17740146). Y. S. is supported by the Japan Society for the Promotion of Science for Young Scientists (No.0509241). T. Y. is supported in part by The 21st Century COE Program "Towards a New Basic Science; Depth and Synthesis".

A. KK expansion with boundary masses

In this appendix, we review derivations of the KK spectra and profiles in a general setup with boundary masses. The 5D metric is given by

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu} + dy^2, \tag{A.1}$$

where $M, N = 0, 1, 2, 3, 5$ are 5D indices, $\mu, \nu = 0, 1, 2, 3$ are 4D ones, and $y \equiv x^5$. The warp factor $\sigma(y)$ is assumed to be a monotonic and nondecreasing function of y and $\sigma(0) = 0$.

A.1 Gauge sector

A 5D gauge multiplet consists of a gauge-scalar Σ, a gauge field A_M, and gauginos λ^i, where $i = 1, 2$ is the $\mathrm{SU}(2)_R$ index. Each field is matrix-valued, i.e.,

$$A_M = \sum_A A_M^A T^A, \tag{A.2}$$

where T^A is a generator of the gauge group. The 5D Lagrangian is written in the 4D $\mathcal{N} = 1$ superspace by introducing the following $\mathcal{N} = 1$ superfields [39, 21],[23]

$$\begin{aligned} V &\equiv -\theta\sigma^\mu\bar{\theta}A_\mu + ie^{\frac{3}{2}\sigma}\theta^2\bar{\theta}\bar{\lambda}^1 - ie^{\frac{3}{2}\sigma}\bar{\theta}^2\theta\lambda^1 + \frac{1}{2}\theta^2\bar{\theta}^2 D, \\ \Phi &\equiv \frac{1}{2}(\Sigma + iA_y) + e^{\frac{1}{2}\sigma}\theta\lambda^2 + \theta^2 F_\Phi, \end{aligned} \tag{A.3}$$

where D and F_Φ are auxiliary fields. We focus on a simple case that the orbifold projection does not break the gauge group at all. Namely, the orbifold parity is assigned as

$$V\ (+,+), \quad \Phi\ (-,-). \tag{A.4}$$

The left (right) signs denote the parities at $y = 0$ ($y = \pi R$).

The 5D Lagrangian is expressed as

$$\mathcal{L}^{\text{gauge}} = \left[\int d^2\theta\, \frac{1}{2g_5^2}\mathrm{tr}\left(\mathcal{W}^\alpha\mathcal{W}_\alpha\right) + \text{h.c.}\right] + e^{-2\sigma}\int d^4\theta\, \frac{1}{g_5^2}\mathrm{tr}\left(\mathcal{V}_5^2\right), \tag{A.5}$$

where g_5 is the 5D gauge coupling, $\mathcal{W}_\alpha$ and $\mathcal{V}_5$ are the gauge-covariant quantities defined as

$$\begin{aligned} \mathcal{W}_\alpha &\equiv \frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V} = -\frac{1}{4}\bar{D}^2 D_\alpha V + \cdots, \\ \mathcal{V}_5 &\equiv e^V\partial_y e^{-V} + \Phi + e^V\Phi^\dagger e^{-V} = -\partial_y V + \Phi + \Phi^\dagger + \cdots, \end{aligned} \tag{A.6}$$

where the ellipses denote quadratic and higher terms. The normalization of the generators are taken as

$$\mathrm{tr}\left(T^A T^B\right) = \frac{1}{2}\delta^{AB}. \tag{A.7}$$

We introduce 4D chiral multiplets ϕ_0^I and ϕ_π^J localized at the orbifold boundaries at $y = 0$ and πR, respectively. The indices I, J run over different irreducible representations of the gauge group. They interact with the 5D gauge multiplet as

$$\mathcal{L}^{\text{bd}} = e^{-2\sigma}\int d^4\theta\, \left\{\sum_I \phi_0^{I\dagger}e^{-V}\phi_0^I\delta(y) + \sum_J \phi_\pi^{J\dagger}e^{-V}\phi_\pi^J\delta(y - \pi R)\right\} + \cdots, \tag{A.8}$$

where the ellipsis denotes the self-interaction terms of $\phi_{0,\pi}$.

[23] For the extension to the 5D SUGRA, see ref. [40]. We will follow the notation of ref. [41].

The above Lagrangians are invariant under the (super-) gauge transformation,

$$
\begin{aligned}
e^V &\to e^{\Lambda} e^V e^{\Lambda^\dagger}, \\
\Phi &\to e^{\Lambda} \left(\Phi - \partial_y\right) e^{-\Lambda}, \\
\phi_0^I &\to e^{\Lambda(0)} \phi_0^I, \qquad\qquad \phi_\pi^J \to e^{\Lambda(\pi R)} \phi_\pi^J.
\end{aligned} \tag{A.9}
$$

The transformation parameter Λ is a chiral superfield. Under this transformation, the gauge-covariant quantities transform as

$$
\begin{aligned}
\mathcal{W}_\alpha &\to e^{\Lambda} \mathcal{W}_\alpha e^{-\Lambda}, \\
\mathcal{V}_5 &\to e^{\Lambda} \mathcal{V}_5 e^{-\Lambda}.
\end{aligned} \tag{A.10}
$$

By choosing the gauge parameter Λ as

$$
\exp\{\Lambda(x,y)\} = \mathcal{P} \exp\left\{ -\int_0^y dy' \; \Phi(x,y') \right\}, \tag{A.11}
$$

we move into the gauge where $\Phi = 0$. The symbol $\mathcal{P}$ stands for the path ordering operator from left to right. Recall that all (non-zero) KK modes of A_y are absorbed into those of A_μ by the "Higgs mechanism", and the latter obtain the KK masses. Thus we can call it unitary gauge. Note that V is no longer in the Wess-Zumino gauge, and its lowest and the next lowest components for $\theta, \bar{\theta}$ are physical degrees of freedom.

Now we assume that the scalar components of ϕ_0^I and ϕ_π^J get VEVs and break the gauge group to a subgroup at the boundaries. Then the Lagrangian becomes

$$
\begin{aligned}
\mathcal{L} = &\left[\int d^2\theta \, \frac{1}{2g_5^2} \mathrm{tr}\left(\mathcal{W}^\alpha \mathcal{W}_\alpha\right) + \mathrm{h.c.} \right] + \frac{e^{-2\sigma}}{g_5^2} \int d^4\theta \; \mathrm{tr}\left\{ (-\partial_y V)^2 + \cdots \right\} \\
&+ \frac{e^{-2\sigma}}{2g_5^2} \int d^4\theta \; \left\{ \sum_{I,A,B} \mathcal{M}_0^{(I)AB} V^A V^B \delta(y) + \sum_{J,A,B} \mathcal{M}_\pi^{(J)AB} V^A V^B \delta(y - \pi R) + \cdots \right\},
\end{aligned} \tag{A.12}
$$

where $\mathcal{M}_0^{(I)AB}$ and $\mathcal{M}_\pi^{(J)AB}$ are the boundary mass parameters defined by

$$
\mathcal{M}_0^{(I)AB} \equiv g_5^2 \langle \phi_0^I \rangle^\dagger T^A T^B \langle \phi_0^I \rangle, \qquad \mathcal{M}_\pi^{(J)AB} \equiv g_5^2 \langle \phi_\pi^J \rangle^\dagger T^A T^B \langle \phi_\pi^J \rangle. \tag{A.13}
$$

The ellipses in eq. (A.12) shows the terms involving the fluctuation around the VEVs, which decouple in the limit of $\mathcal{M}_{0,\pi} \to \infty$. Here we have assumed that SUSY is preserved when $\phi_{0,\pi}$ get the VEVs.

In the following discussions, we consider a case of $\mathcal{M}_0^{(I)AB} = 0$. Then we can always diagonalize the matrix $\sum_J \mathcal{M}_\pi^{(J)AB}$ for the indices A, B by using the gauge symmetry, i.e., $\sum_J \mathcal{M}_\pi^{(J)AB} = \mathcal{M}_A \delta^{AB}$. Thus eq. (A.12) becomes

$$
\begin{aligned}
\mathcal{L} = &\left[\frac{1}{4g_5^2} \int d^2\theta \; \sum_A \mathcal{W}^{A\alpha} \mathcal{W}_\alpha^A + \mathrm{h.c.} \right] \\
&- \frac{1}{2g_5^2} \int d^4\theta \; \sum_A V^A \left\{ \partial_y \left(e^{-2\sigma} \partial_y V^A \right) - e^{-2\sigma} \mathcal{M}_A V^A \delta(y - \pi R) + \cdots \right\},
\end{aligned} \tag{A.14}
$$

where we have performed the partial integration. Now we expand the 5D superfield V into 4D KK modes,

$$V^A(x,y,\theta,\bar{\theta}) = \sum_n f_n^A(y) V_n^A(x,\theta,\bar{\theta}). \tag{A.15}$$

The mode equation for the KK modes is read off from eq. (A.14) as

$$\partial_y\left(e^{-2\sigma}\partial_y f_n^A\right) - e^{-2\sigma}\mathcal{M}_A f_n^A \delta(y-\pi R) = -m_{A,n}^2 f_n^A. \tag{A.16}$$

Since V^A is a Z_2-even superfield, the mode functions $f_n^A(y)$ are even functions around $y = 0, \pi R$. Thus from eq. (A.16), we obtain the following BCs.

$$\begin{aligned} \partial_y f_n^A|_{y=0} &= 0, \\ \left[\partial_y f_n^A\right]_{\pi R-\epsilon}^{\pi R+\epsilon} &= \mathcal{M}_A f_n^A(\pi R). \end{aligned} \tag{A.17}$$

The first condition is the ordinary Neumann BC while the second one is a mixed-type BC. In fact the latter is reduced to the Neumann BC in the limit of $\mathcal{M}_A \to 0$, and it becomes the Dirichlet BC in the limit of $\mathcal{M}_A \to \infty$.

The general solution of eq. (A.16) is written as

$$f_n^A(y) = \alpha_n^A C(y, m_{a,n}) + \beta_n^A S(y, m_{a,n}), \tag{A.18}$$

where α_n^A and β_n^A are real constants determined by the BCs. The functions $C(y,m)$ and $S(y,m)$ are defined in appendix B. The first condition in eq. (A.17) means $\beta_n^A = 0$. So the second condition is translated into

$$-2C'(\pi R, m_{a,n}) = \mathcal{M}_A C(\pi R, m_{a,n}), \tag{A.19}$$

where the prime denotes the y-derivative. This determines the mass spectrum $\{m_{a,n}\}$. The remaining constant α_n^A is fixed by the normalization condition,

$$\int_0^{\pi R} dy\ \left(f_n^A(y)\right)^2 = 1. \tag{A.20}$$

A.2 Hypermultiplet sector

Next we consider a matter sector of hypermultiplets $(\boldsymbol{H}_i, \boldsymbol{H}_i^c)$, where $\boldsymbol{H}_i$ and $\boldsymbol{H}_i^c$ are $\mathcal{N} = 1$ chiral superfields and belong to conjugate representations of the gauge group. Namely, under the gauge transformation eq. (A.9), they transform as

$$\boldsymbol{H}_i \to e^{\Lambda}\boldsymbol{H}_i, \qquad \boldsymbol{H}_i^c \to \boldsymbol{H}_i^c e^{-\Lambda}. \tag{A.21}$$

The index i runs over the irreducible representations. The bulk Lagrangian of this sector is given by

$$\begin{aligned} \mathcal{L}_{\text{bulk}}^{\text{hyper}} &= e^{-2\sigma}\int d^4\theta \sum_i \left(\boldsymbol{H}_i^\dagger e^{-V}\boldsymbol{H}_i + \boldsymbol{H}_i^c e^{V}\boldsymbol{H}_i^{c\dagger}\right) \\ &+ e^{-3\sigma}\left[\int d^2\theta \sum_i \left\{\frac{1}{2}\left(\boldsymbol{H}_i^c\partial_y\boldsymbol{H}_i - \partial_y\boldsymbol{H}_i^c\boldsymbol{H}_i\right)\right.\right. \\ &\qquad\qquad \left.\left. - \boldsymbol{H}_i^c\Phi\boldsymbol{H}_i + M_i\varepsilon(y)\boldsymbol{H}_i^c\boldsymbol{H}_i\right\} + \text{h.c.}\right], \end{aligned} \tag{A.22}$$

where M_i's are bulk mass parameters and $\varepsilon(y)$ is the periodic step function.

For simplicity, let us focus on two hypermultiplets among $(\boldsymbol{H}_i, \boldsymbol{H}_i^c)$, and denote them as (h, h^c) and (H, H^c). They are components either in the same gauge multiplet or different one. Then the Lagrangian for them is written as

$$\begin{aligned}\mathcal{L}_{\rm bulk}^{\rm hyper} &= e^{-2\sigma}\int d^4\theta\left\{|h|^2+|h^c|^2+|H|^2+|H^c|^2\right\}\\ &\quad +e^{-3\sigma}\left[\int d^2\theta\left\{\frac{1}{2}\left(h^c\partial_y h - h\partial_y h^c + H^c\partial_y H - H\partial_y H^c\right)\right.\right.\\ &\quad\left.\left. + M_h\varepsilon(y)h^c h + M_H\varepsilon(y)H^c H\right\} + \text{h.c.}\right] + \cdots, \qquad (A.23)\end{aligned}$$

where M_h and M_H are the bulk mass parameters. In the case that (h, h^c) and (H, H^c) belong to the same gauge multiplet, $M_h = M_H$.

A.2.1 Boundary mass terms

Here we consider effects from mass terms localized at $y = \pi R$, which are induced by the VEVs of ϕ_π^J. (We do not consider the boundary masses coming from ϕ_0^I, for simplicity.) Each chiral superfield has an opposite orbifold parity to the chiral partner (contained in the same hypermultiplet). Thus there are the following three cases according to the orbifold parity assignments.

Case 1. First we consider a case that both the hypermultiplets have the same parities at both the orbifold boundaries, i.e.,

$$h\,(+,+),\quad h^c\,(-,-),\quad H\,(+,+),\quad H^c\,(-,-). \qquad (A.24)$$

The left (right) signs denote the parities at $y = 0$ $(y = \pi R)$. In this case the most general boundary mass terms are given by

$$\mathcal{L}_{\rm bd}^{\rm hyper} = e^{-3\sigma}\left[\int d^2\theta\,\begin{pmatrix} h & H\end{pmatrix}\begin{pmatrix}\kappa & \eta\\ \eta & \lambda\end{pmatrix}\begin{pmatrix} h\\ H\end{pmatrix}\delta(y-\pi R) + \text{h.c.}\right], \qquad (A.25)$$

where the Majorana masses κ, λ and the Dirac mass η are dimensionless parameters.[24] We treat them as complex parameters, although two phases among κ, λ and η can be absorbed by field redefinitions unless the phases of h and H are fixed in another sector.

The mode equations are given by

$$\begin{aligned}-\frac{1}{4}\bar{D}^2\bar{h} - e^{-\sigma}\left(\partial_y - \frac{3}{2}\sigma' - M_h\varepsilon\right)h^c + 2e^{-\sigma}(\kappa h + \eta H)\delta(y-\pi R) &= 0,\\ -\frac{1}{4}\bar{D}^2\bar{h}^c + e^{-\sigma}\left(\partial_y - \frac{3}{2}\sigma' + M_h\varepsilon\right)h &= 0,\\ -\frac{1}{4}\bar{D}^2\bar{H} - e^{-\sigma}\left(\partial_y - \frac{3}{2}\sigma' - M_H\varepsilon\right)H^c + 2e^{-\sigma}(\eta h + \lambda H)\delta(y-\pi R) &= 0,\\ -\frac{1}{4}\bar{D}^2\bar{H}^c + e^{-\sigma}\left(\partial_y - \frac{3}{2}\sigma' + M_H\varepsilon\right)H &= 0. \qquad (A.26)\end{aligned}$$

[24]These parameters correspond to ratios of the boundary Higgs VEVs to the 5D cutoff scale Λ_*.

The BCs at $y=0$ are determined only by the orbifold parities[25] as

$$h^c\big|_{y=0} = H^c\big|_{y=0} = 0, \tag{A.27}$$

since there are no boundary terms there. On the other hand, the BCs at $y = \pi R$ are modified by the boundary mass terms as

$$\begin{aligned}
[h^c]_{\pi R-\epsilon}^{\pi R+\epsilon} &= 2\left\{\kappa h + \eta H\right\}_{y=\pi R}, \\
[H^c]_{\pi R-\epsilon}^{\pi R+\epsilon} &= 2\left\{\eta h + \lambda H\right\}_{y=\pi R}.
\end{aligned} \tag{A.28}$$

Now we expand the 5D superfields into 4D KK modes as

$$\begin{aligned}
h(x,y,\theta) &= \sum_n e^{\frac{3}{2}\sigma} f_{h,n}(y) h_n(x,\theta), \qquad & h^c(x,y,\theta) &= \sum_n e^{\frac{3}{2}\sigma} f^c_{h,n}(y) h^c_n(x,\theta), \\
H(x,y,\theta) &= \sum_n e^{\frac{3}{2}\sigma} f_{H,n}(y) H_n(x,\theta), \qquad & H^c(x,y,\theta) &= \sum_n e^{\frac{3}{2}\sigma} f^c_{H,n}(y) H^c_n(x,\theta).
\end{aligned} \tag{A.29}$$

The mode equations in the bulk ($0 < y < \pi R$) are given by

$$\begin{aligned}
(\partial_y + M_\phi) f_{\phi,n} &= m_n e^\sigma f^{c*}_{\phi,n}, \\
(\partial_y - M_\phi) f^c_{\phi,n} &= -m_n e^\sigma f^{*}_{\phi,n},
\end{aligned} \tag{A.30}$$

where $\phi = h, H$. From the BCs in eq. (A.28), the mode functions must satisfy the conditions,

$$f^c_{h,n}(0) = f^c_{H,n}(0) = 0, \tag{A.31}$$

$$\begin{aligned}
f^c_{h,n}(\pi R - \epsilon) &= -\kappa f_{h,n}(\pi R) - \eta f_{H,n}(\pi R), \\
f^c_{H,n}(\pi R - \epsilon) &= -\eta f_{h,n}(\pi R) - \lambda f_{H,n}(\pi R).
\end{aligned} \tag{A.32}$$

Solutions of eq. (A.30) with the BC in eq. (A.31) are given by

$$\begin{aligned}
f_{\phi,n}(y) &= \alpha_{\phi,n} e^{-M_\phi y} C_{M_\phi}(y, m_n), \\
f^c_{\phi,n}(y) &= -\alpha^*_{\phi,n} e^{M_\phi y} S_{-M_\phi}(y, m_n),
\end{aligned} \tag{A.33}$$

where $\phi = h, H$, and $(\alpha_{h,n}, \alpha_{H,n})$ are complex constants determined by the BCs. The functions $C_M(y,m)$ and $S_M(y,m)$ are defined in appendix B. Thus the BCs in eq. (A.32) are rewritten as

$$\mathcal{M}_4 \begin{pmatrix} \mathrm{Re}\,\alpha_{h,n} \\ \mathrm{Re}\,\alpha_{H,n} \\ \mathrm{Im}\,\alpha_{h,n} \\ \mathrm{Im}\,\alpha_{H,n} \end{pmatrix} = 0, \tag{A.34}$$

with

$$\mathcal{M}_4 \equiv \begin{pmatrix} \kappa_R \tilde{C}_{M_h} - \tilde{S}_{-M_h} & \eta_R \tilde{C}_{M_H} & -\kappa_I \tilde{C}_{M_h} & -\eta_I \tilde{C}_{M_H} \\ \eta_R \tilde{C}_{M_h} & \lambda_R \tilde{C}_{M_H} - \tilde{S}_{-M_H} & -\eta_I \tilde{C}_{M_h} & -\lambda_I \tilde{C}_{M_H} \\ \kappa_I \tilde{C}_{M_h} & \eta_I \tilde{C}_{M_H} & \kappa_R \tilde{C}_{M_h} + \tilde{S}_{-M_h} & \eta_R \tilde{C}_{M_H} \\ \eta_I \tilde{C}_{M_h} & \lambda_I \tilde{C}_{M_H} & \eta_R \tilde{C}_{M_h} & \lambda_R \tilde{C}_{M_H} + \tilde{S}_{-M_H} \end{pmatrix}, \tag{A.35}$$

[25] The BCs for h and H do not provide independent informations from those for h^c and H^c, because the former mode functions are related to the latter ones through the bulk equations of motion. (*See* eq. (A.30).)

where

$$\tilde{S}_{-M} \equiv e^{M\pi R} S_{-M}(\pi R, m_n), \qquad \tilde{C}_M \equiv e^{-M\pi R} C_M(\pi R, m_n), \tag{A.36}$$

and $\kappa_R \equiv \mathrm{Re}\,\kappa$, $\kappa_I \equiv \mathrm{Im}\,\kappa$, and so on. The condition that eq. (A.34) has a nontrivial solution is

$$\det \mathcal{M}_4 = \tilde{C}^2_{M_h} \tilde{C}^2_{M_H} \left(\tilde{T}^2_{M_h} \tilde{T}^2_{M_H} - 2\,|\eta|^2\, \tilde{T}_{M_h} \tilde{T}_{M_H} - |\lambda|^2\, \tilde{T}^2_{M_h} - |\kappa|^2\, \tilde{T}^2_{M_H} + \left|\kappa\lambda - \eta^2\right|^2 \right) = 0, \tag{A.37}$$

where

$$\tilde{T}_M \equiv \frac{\tilde{S}_{-M}}{\tilde{C}_M}. \tag{A.38}$$

Equation (A.37) determines the mass spectrum. The complex constants $\alpha_{h,n}$ and $\alpha_{H,n}$ are determined by eq. (A.34) with the solution of eq. (A.37), and the normalization condition,

$$\int_0^{\pi R} dy\ \left\{ |f_{h,n}(y)|^2 + |f_{H,n}(y)|^2 \right\} = 1. \tag{A.39}$$

Case 2. Next we consider a case that one hypermultiplet has the same parities at both boundaries while the other has opposite parities, i.e.,

$$h\,(+,-), \quad h^c\,(-,+), \quad H\,(+,+), \quad H^c\,(-,-). \tag{A.40}$$

In this case the most general boundary mass terms are given by

$$\mathcal{L}^{\mathrm{hyper}}_{\mathrm{bd}} = e^{-3\sigma} \left[\int d^2\theta\ \begin{pmatrix} h^c & H \end{pmatrix} \begin{pmatrix} \kappa & \eta \\ \eta & \lambda \end{pmatrix} \begin{pmatrix} h^c \\ H \end{pmatrix} \delta(y - \pi R) + \mathrm{h.c.} \right], \tag{A.41}$$

where the dimensionless mass parameters κ, λ and η are complex. Through similar calculations to the Case 1, we obtain an equation that determines the mass spectrum. It corresponds to eq. (A.37) with the replacement of

$$(\tilde{S}_{-M_h}, \tilde{C}_{M_h}) \to (\tilde{C}_{M_h}, -\tilde{S}_{-M_h}). \tag{A.42}$$

Namely, the mass spectrum is determined by

$$\det \mathcal{M}_4 = \tilde{S}^2_{-M_h} \tilde{C}^2_{M_H} \left(\tilde{T}^{-2}_{M_h} \tilde{T}^2_{M_H} + 2\,|\eta|^2\, \tilde{T}^{-1}_{M_h} \tilde{T}_{M_H} - |\lambda|^2\, \tilde{T}^{-2}_{M_h} - |\kappa|^2\, \tilde{T}^2_{M_H} + \left|\kappa\lambda - \eta^2\right|^2 \right) = 0. \tag{A.43}$$

Case 3. Finally we consider a case that both the hypermultiplets have opposite parities at the two boundaries, i.e.,

$$h\,(+,-), \quad h^c\,(-,+), \quad H\,(+,-), \quad H^c\,(-,+). \tag{A.44}$$

In this case the most general boundary mass terms are given by

$$\mathcal{L}^{\mathrm{hyper}}_{\mathrm{bd}} = e^{-3\sigma} \left[\int d^2\theta\ \begin{pmatrix} h^c & H^c \end{pmatrix} \begin{pmatrix} \kappa & \eta \\ \eta & \lambda \end{pmatrix} \begin{pmatrix} h^c \\ H^c \end{pmatrix} \delta(y - \pi R) + \mathrm{h.c.} \right], \tag{A.45}$$

where the dimensionless mass parameters κ, λ and η are complex. The equation that determines the mass spectrum corresponds to eq. (A.37) with the replacement,

$$\begin{aligned}(\tilde{S}_{-M_h}, \tilde{C}_{M_h}) &\to (\tilde{C}_{M_h}, -\tilde{S}_{-M_h}),\\ (\tilde{S}_{-M_H}, \tilde{C}_{M_H}) &\to (\tilde{C}_{M_H}, -\tilde{S}_{-M_H}).\end{aligned} \tag{A.46}$$

Thus, the mass spectrum is determined by

$$\det \mathcal{M}_4 = \tilde{S}^2_{-M_h}\tilde{S}^2_{-M_H}\left(\tilde{T}^{-2}_{M_h}\tilde{T}^{-2}_{M_H} - 2\,|\eta|^2\,\tilde{T}^{-1}_{M_h}\tilde{T}^{-1}_{M_H} - |\lambda|^2\,\tilde{T}^{-2}_{M_h} - |\kappa|^2\,\tilde{T}^{-2}_{M_H} + \left|\kappa\lambda - \eta^2\right|^2\right) = 0. \tag{A.47}$$

A.2.2 Mixing with boundary fields

Now let us consider effects from brane mass terms between a bulk hypermultiplet (H, H^c) and a 4D chiral superfield χ localized on the $y = \pi R$ brane.

Case 4. First we consider a case that the hypermultiplet has the same parities at both boundaries, i.e.,

$$H\ (+,+), \quad H^c\ (-,-). \tag{A.48}$$

The boundary Lagrangian in this case is

$$\mathcal{L}_{\rm bd} = \left\{e^{-2\sigma(\pi R)}\int d^4\theta\ |\chi|^2 + e^{-3\sigma(\pi R)}\left[\int d^2\theta\ \left(\xi H\chi + \frac{1}{2}m_\chi\chi^2\right) + \text{h.c.}\right]\right\}\delta(y - \pi R). \tag{A.49}$$

The constants ξ and m_χ have mass-dimension 1/2 and 1, respectively. The equations of motion are

$$\begin{aligned} -\frac{1}{4}\bar{D}^2\bar{H} + e^{-\sigma}\left\{-\left(\partial_y - \frac{3}{2}\sigma' - M_H\varepsilon\right)H^c + \xi\chi\delta(y - \pi R)\right\} &= 0,\\ -\frac{1}{4}\bar{D}^2\bar{H}^c + e^{-\sigma}\left(\partial_y - \frac{3}{2}\sigma' + M_H\varepsilon\right)H &= 0,\\ -\frac{1}{4}\bar{D}^2\bar{\chi} + e^{-\sigma(\pi R)}\left\{\xi H|_{y=\pi R} + m_\chi\chi\right\} &= 0, \end{aligned} \tag{A.50}$$

From the first equation, we obtain a relation between the boundary and bulk superfields as

$$[H^c]^{\pi R+\epsilon}_{\pi R-\epsilon} = \xi\chi. \tag{A.51}$$

Using this relation, the last equation in eq. (A.50) is rewritten as

$$\left[-\frac{1}{4}\bar{D}^2\bar{H}^c - e^{-\sigma}\left(\frac{|\xi|^2}{2}H - \hat{m}_\chi H^c\right)\right]_{y=\pi R-\epsilon} = 0, \tag{A.52}$$

where $\hat{m}_\chi \equiv m_\chi\bar{\xi}/\xi$. Thus the mode functions satisfy the following BC,

$$\frac{|\xi|^2}{2}f_{H,n}(\pi R) - \hat{m}_\chi f^c_{H,n}(\pi R - \epsilon) = -e^{\sigma(\pi R)}m_n f^{c*}_{H,n}(\pi R - \epsilon). \tag{A.53}$$

This is translated by using eq. (A.33) into

$$\left(\frac{|\xi|^2}{2} \tilde{C}_{M_H} - e^{\sigma(\pi R)} m_n \tilde{S}_{-M_H} \right) \alpha_{H,n} + \hat{m}_\chi \tilde{S}_{-M_H} \alpha^*_{H,n} = 0. \tag{A.54}$$

The condition for it to have a nontrivial solution is

$$\left(\frac{|\xi|^2}{2} \tilde{C}_{M_H} - e^{\sigma(\pi R)} m_n \tilde{S}_{-M_H} \right)^2 - \left| \hat{m}_\chi \tilde{S}_{-M_H} \right|^2 = 0, \tag{A.55}$$

or

$$\tilde{T}_{M_H} = \frac{|\xi|^2}{2\left(e^{\sigma(\pi R)} m_n \pm |m_\chi| \right)}. \tag{A.56}$$

Case 5. Next we consider a case that the hypermultiplet has opposite parities at both boundaries, i.e.,

$$H\ (+,-), \quad H^c\ (-,+). \tag{A.57}$$

The boundary Lagrangians in this case are

$$\mathcal{L}_{\rm bd} = \left\{ e^{-2\sigma(\pi R)} \int d^4\theta\ |\chi|^2 + e^{-3\sigma(\pi R)} \left[\int d^2\theta\ \left(\xi H^c \chi + \frac{1}{2} m_\chi \chi^2 \right) + \text{h.c.} \right] \right\} \delta(y - \pi R). \tag{A.58}$$

where ξ and m_χ are complex parameters whose mass-dimensions are 1/2 and 1. Through the similar calculations to the Case 4, we obtain an equation that determines the mass spectrum. It is obtained from eq. (A.56) by the replacement,

$$(\tilde{S}_{\pm M_H}, \tilde{C}_{\pm M_H}) \to (\tilde{C}_{\mp M_H}, -\tilde{S}_{\mp M_H}). \tag{A.59}$$

Then the mass spectrum is determined by

$$\tilde{T}^{-1}_{M_H} = -\frac{|\xi|^2}{2\left(e^{\sigma(\pi R)} m_n \pm |m_\chi| \right)}. \tag{A.60}$$

B. Bases of mode functions

This section shows the definitions and properties of functions, $C(y,m)$, $S(y,m)$, $C_M(y,m)$, and $S_M(y,m)$, following ref. [42]. The functions $C(y,m)$ and $S(y,m)$ are defined as solutions of

$$\left\{ \partial_y^2 - 2\sigma' \partial_y + m^2 e^{2\sigma} \right\} f = 0 \tag{B.1}$$

with initial conditions of

$$\begin{aligned} C(0,m) &= 1, & C'(0,m) &= 0, \\ S(0,m) &= 0, & S'(0,m) &= m. \end{aligned} \tag{B.2}$$

From the Wronskian relation, they satisfy

$$S'(y,m)C(y,m) - C'(y,m)S(y,m) = m e^{2\sigma(y)}. \tag{B.3}$$

Next we provide the definition of $C_M(y,m)$ and $S_M(y,m)$. Combining the two equations in eq. (A.30), we obtain the following type of the second order differential equation,

$$\left\{\partial_y^2 - \sigma'\partial_y - M(M+\sigma') + m^2 e^{2\sigma}\right\} f_M = 0. \tag{B.4}$$

By redefining $f_M(y)$ as

$$\tilde{f}_M(y) \equiv e^{My} f_M(y), \tag{B.5}$$

eq. (B.4) becomes

$$\left\{\partial_y^2 - (\sigma' + 2M)\partial_y + m^2 e^{2\sigma}\right\} \tilde{f}_M = 0. \tag{B.6}$$

The functions $C_M(y,m)$ and $S_M(y,m)$ are solutions of eq. (B.6) which satisfy initial conditions,

$$\begin{aligned} C_M(0,m) &= 1, & C'_M(0,m) &= 0, \\ S_M(0,m) &= 0, & S'_M(0,m) &= m. \end{aligned} \tag{B.7}$$

Here let us define a function $\tilde{g}_M(y)$ as

$$\tilde{g}_M(y) \equiv e^{-\sigma(y)-2My} \tilde{f}'_M(y). \tag{B.8}$$

Then it satisfies

$$\left\{\partial_y^2 - (\sigma' - 2M)\partial_y + m^2 e^{2\sigma}\right\} \tilde{g}_M = 0. \tag{B.9}$$

This means $\tilde{g}_M(y) \propto \tilde{f}_{-M}(y)$. Taking into account the initial conditions, we obtain

$$\begin{aligned} C'_M(y,m) &= -m e^{\sigma+2My} S_{-M}(y,m), \\ S'_M(y,m) &= m e^{\sigma+2My} C_{-M}(y,m). \end{aligned} \tag{B.10}$$

Furthermore, using the Wronskian relation, we obtain

$$S'_M(y,m) C_M(y,m) - C'_M(y,m) S_M(y,m) = m e^{\sigma(y)+2My}, \tag{B.11}$$

which is translated into

$$C_M(y,m) C_{-M}(y,m) + S_M(y,m) S_{-M}(y,m) = 1, \tag{B.12}$$

by using eq. (B.10).

B.1 Flat spacetime

Let us see explicit forms of $C_M(y,m)$ and $S_M(y,m)$ in the case of the flat spacetime, i.e., $\sigma(y) = 0$. In this case, eq. (B.1) is reduced to

$$\left(\partial_y^2 + m^2\right) f = 0, \tag{B.13}$$

and

$$C(y,m) = \cos(my), \qquad S(y,m) = \sin(my). \tag{B.14}$$

On the other hand, eq. (B.4) becomes

$$\left(\partial_y^2 - M^2 + m^2\right) f_M = 0. \tag{B.15}$$

Thus a general solution is given by

$$f_M(y) = A\cos\left(\sqrt{m^2-M^2}y\right) + B\sin\left(\sqrt{m^2-M^2}y\right), \tag{B.16}$$

where A and B are integration constants, and $m^2 \geq M^2$ is assumed. From eqs. (B.5) and (B.7), we obtain

$$\begin{aligned} C_M(y,m) &= e^{My}\left\{\cos\left(\sqrt{m^2-M^2}y\right) - \frac{M}{\sqrt{m^2-M^2}}\sin\left(\sqrt{m^2-M^2}y\right)\right\} \\ &= \frac{m}{\sqrt{m^2-M^2}}e^{My}\cos\left(\sqrt{m^2-M^2}y+\varphi\right), \\ S_M(y,m) &= \frac{m}{\sqrt{m^2-M^2}}e^{My}\sin\left(\sqrt{m^2-M^2}y\right), \end{aligned} \tag{B.17}$$

where

$$\varphi \equiv \arctan\left(\frac{M}{\sqrt{m^2-M^2}}\right). \tag{B.18}$$

B.2 Randall-Sundrum spacetime

Finally we consider the case of the warped background setup of $\sigma(y) = ky$ ($k>0$, $0 \leq y \leq \pi R$). In this case solutions of eq. (B.6) are expressed by the Bessel functions as

$$\begin{aligned} C_M(y,m) &= \frac{\pi m}{2k}e^{\alpha ky}\left\{Y_{\alpha-1}\left(\frac{m}{k}\right)J_\alpha\left(\frac{m}{k}e^{ky}\right) - J_{\alpha-1}\left(\frac{m}{k}\right)Y_\alpha\left(\frac{m}{k}e^{ky}\right)\right\} \\ &= \frac{\pi m}{2k}\frac{e^{\alpha ky}}{\sin\pi\alpha}\left\{J_{1-\alpha}\left(\frac{m}{k}\right)J_\alpha\left(\frac{m}{k}e^{ky}\right) + J_{\alpha-1}\left(\frac{m}{k}\right)J_{-\alpha}\left(\frac{m}{k}e^{ky}\right)\right\}, \\ S_M(y,m) &= -\frac{\pi m}{2k}e^{\alpha ky}\left\{Y_\alpha\left(\frac{m}{k}\right)J_\alpha\left(\frac{k}{m}e^{ky}\right) - J_\alpha\left(\frac{m}{k}\right)Y_\alpha\left(\frac{m}{k}e^{ky}\right)\right\} \\ &= \frac{\pi m}{2k}\frac{e^{\alpha ky}}{\sin\pi\alpha}\left\{J_{-\alpha}\left(\frac{m}{k}\right)J_\alpha\left(\frac{m}{k}e^{ky}\right) - J_\alpha\left(\frac{m}{k}\right)J_{-\alpha}\left(\frac{m}{k}e^{ky}\right)\right\}, \end{aligned} \tag{B.19}$$

where $\alpha \equiv (M/k) + 1/2$. We can see

$$C(y,m) = C_{k/2}(y,m), \qquad S(y,m) = S_{k/2}(y,m), \tag{B.20}$$

from eqs. (B.1) and (B.6).

References

[1] Particle Data Group collaboration, W.M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[2] Y. Kawamura, *Triplet-doublet splitting, proton stability and extra dimension*, *Prog. Theor. Phys.* **105** (2001) 999 [hep-ph/0012125]; *Split multiplets, coupling unification and extra dimension*, *Prog. Theor. Phys.* **105** (2001) 691 [hep-ph/0012352].

[3] L.J. Hall and Y. Nomura, *Gauge unification in higher dimensions*, *Phys. Rev.* **D 64** (2001) 055003 [hep-ph/0103125].

[4] L.J. Hall and Y. Nomura, *Gauge coupling unification from unified theories in higher dimensions*, *Phys. Rev.* **D 65** (2002) 125012 [hep-ph/0111068].

[5] H. Murayama and A. Pierce, *Not even decoupling can save minimal supersymmetric* SU(5), *Phys. Rev.* **D 65** (2002) 055009 [hep-ph/0108104].

[6] D.B. Fairlie, *Higgs' fields and the determination of the Weinberg angle*, *Phys. Lett.* **B 82** (1979) 97; *Two consistent calculations of the Weinberg angle*, *J. Phys.* **G 5** (1979) L55;
N.S. Manton, *A new six-dimensional approach to the Weinberg-Salam model*, *Nucl. Phys.* **B 158** (1979) 141;
P. Forgacs and N.S. Manton, *Space-time symmetries in gauge theories*, *Commun. Math. Phys.* **72** (1980) 15.

[7] R. Dermisek and A. Mafi, SO(10) *grand unification in five dimensions: proton decay and the mu problem*, *Phys. Rev.* **D 65** (2002) 055002 [hep-ph/0108139].

[8] T. Asaka, W. Buchmüller and L. Covi, *Gauge unification in six dimensions*, *Phys. Lett.* **B 523** (2001) 199 [hep-ph/0108021];
L.J. Hall, Y. Nomura, T. Okui and D.R. Smith, SO(10) *unified theories in six dimensions*, *Phys. Rev.* **D 65** (2002) 035008 [hep-ph/0108071].

[9] C. Csáki, C. Grojean, H. Murayama, L. Pilo and J. Terning, *Gauge theories on an interval: unitarity without a Higgs*, *Phys. Rev.* **D 69** (2004) 055006 [hep-ph/0305237].

[10] N. Sakai and N. Uekusa, *Selecting gauge theories on an interval by* 5*D gauge transformations*, *Prog. Theor. Phys.* **118** (2007) 315 [hep-th/0604121].

[11] T. Ohl and C. Schwinn, *Unitarity, BRST symmetry and Ward identities in orbifold gauge theories*, *Phys. Rev.* **D 70** (2004) 045019 [hep-ph/0312263];
Y. Abe et al., 4*D equivalence theorem and gauge symmetry on orbifold*, *Prog. Theor. Phys.* **113** (2005) 199 [hep-th/0402146].

[12] Y. Nomura, D.R. Smith and N. Weiner, *GUT breaking on the brane*, *Nucl. Phys.* **B 613** (2001) 147 [hep-ph/0104041].

[13] C. Csáki, C. Grojean, L. Pilo and J. Terning, *Towards a realistic model of Higgsless electroweak symmetry breaking*, *Phys. Rev. Lett.* **92** (2004) 101802 [hep-ph/0308038].

[14] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys.* **B 719** (2005) 165 [hep-ph/0412089].

[15] C.D. Carone and J.M. Conroy, *Higgsless GUT breaking and trinification*, *Phys. Rev.* **D 70** (2004) 075013 [hep-ph/0407116].

[16] A. Hebecker and J. March-Russell, *Proton decay signatures of orbifold GUTs*, *Phys. Lett.* **B 539** (2002) 119 [hep-ph/0204037].

[17] D.E. Kaplan, G.D. Kribs and M. Schmaltz, *Supersymmetry breaking through transparent extra dimensions*, *Phys. Rev.* **D 62** (2000) 035010 [hep-ph/9911293];
Z. Chacko, M.A. Luty, A.E. Nelson and E. Ponton, *Gaugino mediated supersymmetry breaking*, *JHEP* **01** (2000) 003 [hep-ph/9911323].

[18] G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [hep-ph/9810442];
L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155].

[19] J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev.* **D 39** (1989) 844;
A. de Gouvêa, A. Friedland and H. Murayama, *Next-to-minimal supersymmetric standard model with the gauge mediation of supersymmetry breaking*, *Phys. Rev.* **D 57** (1998) 5676 [hep-ph/9711264].

[20] L.J. Hall, J.D. Lykken and S. Weinberg, *Supergravity as the messenger of supersymmetry breaking*, *Phys. Rev.* **D 27** (1983) 2359.

[21] D. Marti and A. Pomarol, *Supersymmetric theories with compact extra dimensions in $N = 1$ superfields*, *Phys. Rev.* **D 64** (2001) 105025 [hep-th/0106256].

[22] D.E. Kaplan and N. Weiner, *Radion mediated supersymmetry breaking as a Scherk-Schwarz theory*, hep-ph/0108001.

[23] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60; *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[24] H. Abe and Y. Sakamura, *Scherk-Schwarz SUSY breaking from the viewpoint of 5D conformal supergravity*, *JHEP* **02** (2006) 014 [hep-th/0512326]; *Supersymmetry breaking in a warped slice with Majorana-type masses*, *JHEP* **03** (2007) 106 [hep-th/0702097].

[25] J.A. Bagger, F. Feruglio and F. Zwirner, *Generalized symmetry breaking on orbifolds*, *Phys. Rev. Lett.* **88** (2002) 101601 [hep-th/0107128]; *Brane induced supersymmetry breaking*, *JHEP* **02** (2002) 010 [hep-th/0108010];
C. Biggio, F. Feruglio, A. Wulzer and F. Zwirner, *Equivalent effective Lagrangians for Scherk-Schwarz compactifications*, *JHEP* **11** (2002) 013 [hep-th/0209046].

[26] TITAND Working Group collaboration, Y. Suzuki et al., *Multi-megaton water Cherenkov detector for a proton decay search: TITAND (former name: TITANIC)*, hep-ex/0110005.

[27] Y. Aoki, C. Dawson, J. Noaki and A. Soni, *Proton decay matrix elements with domain-wall fermions*, *Phys. Rev.* **D 75** (2007) 014507 [hep-lat/0607002].

[28] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, *Nucl. Phys.* **B 147** (1979) 277.

[29] N. Arkani-Hamed and M. Schmaltz, *Hierarchies without symmetries from extra dimensions*, *Phys. Rev.* **D 61** (2000) 033005 [hep-ph/9903417].

[30] H. Abe, K. Choi, K.-S. Jeong and K.-i. Okumura, *Scherk-Schwarz supersymmetry breaking for quasi-localized matter fields and supersymmetry flavor violation*, *JHEP* **09** (2004) 015 [hep-ph/0407005].

[31] F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961.

[32] A. Masiero, S.K. Vempati and O. Vives, *Seesaw and lepton flavour violation in SUSY* SO(10), *Nucl. Phys.* **B 649** (2003) 189 [hep-ph/0209303].

[33] G.F. Giudice and A. Masiero, *A natural solution to the μ problem in supergravity theories*, *Phys. Lett.* **B 206** (1988) 480.

[34] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[35] M. Artuso et al., *B, D and K decays*, arXiv:0801.1833.

[36] Y. Hosotani and Y. Sakamura, *Anomalous Higgs couplings in the $SO(5) \times \mathrm{U}(1)_{B-L}$ Gauge-Higgs unification in warped spacetime*, *Prog. Theor. Phys.* **118** (2007) 935 [hep-ph/0703212].

[37] S. Dimopoulos and F. Wilczek, *Incomplete multiplets in supersymmetric unified models*, NSF-ITP-82-07.

[38] L. Randall and M.D. Schwartz, *Unification and the hierarchy from AdS_5*, *Phys. Rev. Lett.* **88** (2002) 081801 [hep-th/0108115].

[39] N. Arkani-Hamed, T. Gregoire and J.G. Wacker, *Higher dimensional supersymmetry in 4D superspace*, *JHEP* **03** (2002) 055 [hep-th/0101233].

[40] F. Paccetti Correia, M.G. Schmidt and Z. Tavartkiladze, *nSuperfield approach to 5D conformal SUGRA and the radion*, *Nucl. Phys.* **B 709** (2005) 141 [hep-th/0408138]; H. Abe and Y. Sakamura, *Superfield description of 5D supergravity on general warped geometry*, *JHEP* **10** (2004) 013 [hep-th/0408224].

[41] J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton University Press, Princeton U.S.A. (1992).

[42] A. Falkowski, *About the holographic pseudo-Goldstone boson*, *Phys. Rev.* **D 75** (2007) 025017 [hep-ph/0610336].

JHEP06(2008)044